Original Article

A comparative analysis of flow features of Newtonian and power law material: A New configuration

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A B S T R A C T

In recent years, material analysis of fluids has generated prodigious interest of researchers due to their effective role in interdisciplinary sciences. In view of its importance, the present communication is devoted to analyze the flow of power law fluid representing the features of shear thinning, shear thickening and Newtonian materials. Constitutive equations expressed in the form of tensorial representations depicting power law relation between viscosity and shear rate. The whole mathematical model is solved computationally via finite element method by using stable $P_2$ – $P_1$ finite element pair. A highly refined hybrid mesh is obliged for the accurate computation of results. Material properties of power law fluid are disclosed in physical configuration renowned as channel driven cavity combining various benchmark problems like cavity flow, forward and backward facing steps and channel flow. Impact of material parameters on pertinent profiles is disclosed through graphs. Verification of computed results is done by comparing the velocity, viscosity, pressure fields for power law fluid with the Newtonian case.

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1. Introduction

Significance of domain in the interpretation of physical attributes of flowing fluid materials cannot be neglected. In recent years physical configuration like cavity has intended the direction of research fraternity towards itself due to its dynamical features and practical utility in multiple disciplines. For example Aidun et al. [1] pointed the direct relevancy of cavity flows to coasters, melting spinning processes, micro-crystalline materials and so forth. Zumbrunnen et al. [2] explained the application of eddies formation in drag reducing riblets and synthesis of fine polymer composites. The
importance of fluid flow in a cavity is acknowledged in a way that it is one of the benchmark problems is the field of computational fluid dynamics and been interrogated under various physical restrictions. For the sake of interest of readers few recent investigations are cited in Refs. [3–6]. Over the years researcher are trying to modify the cavity flow to accommodate it according to new world applications. In this regard an amendment is made by combining the flow in cavity and channel and termed as channel driven cavity. One of the most interesting features of channel driven cavity is that it covers various benchmark problems like flow in cavity and channel, forward and backward facing steps processes, contraction and expansion phenomenon. So the development and fabrication of this configuration will serve as benchmark study for all type of problems and validate the previously computed results.

During the last decades or so, there has been a growing recognition about utilized non-Newtonian materials in industrial sector like aqueous foams, mixture of immiscible materials, slurries for instance and polymers and solutions (man-made and natural) [7–9]. In view of consumption and implications of non-Newtonian materials in engineering applications huge amount of materials their narration is important task. Thus, the rheologists have classified non-Newtonian materials into various categorizations on the basis of diversified nature [10–14]. Among these diversifications shear thinning, shear thickening and rheoplastic materials are on main stream. After working on nature of different non-Newtonian material various fluid models are proposed. Among these models power law model is considered as such a dynamical fluid which possesses features of shear thickening, shear thinning and Newtonian materials. In recent year several though provoking studies regarding the Power law material is executed. Like, Patnana et al. [15] investigated uplift in drag coefficient against fixed magnitude of Reynold number and power law index in incompressible generalized Newtonian (power law) fluid across an unconfined circular cylinder. They measured drag coefficient variance against power law parameter and Reynold number. Chhabra et al. [16] investigated steady and incompressible flow of power law fluid past a circular cylinder by implementing finite difference scheme for different magnitudes of Reynold number 1, 20 and 40. Alessio and Pascal [17] measured variation in hydrodynamic forces against the power-law index and of the Reynolds number. Paliwal et al. [18] worked on power law liquids across square obstacle. Flowing kinematics of Power law fluid across a circular cylinder and solved numerically by utilizing semi implicit finite volume method was explicated by Coelho et al. [19]. Analysis on magnetohydrodynamics flow of non-Newtonian Williamson fluid flow generated by stretching of sheet under the effectiveness of multiple slips was manifested by Raza et al. [20]. They found the solution of problem by way of numerical approach and inferred that maximum friction factor is ascertained at strong magnetic field. Gourari et al. [21] interpreted the flow features imparted by natural convection to water flowing between two coaxial cylinders by implementing finite volume approach. Fateh and Oudina [22] disclosed thermophysical aspects of Cu-water nanofluid in a vertical cylindrical annulus enclosure with two discrete heat sources of different lengths. Fateh [23] performed analytical treatment by way of Homotopy analysis method for magnetohydrodynamic flow of Jeffrey Hamel fluid. Fateh and Oudina [24] probed hydrodynamic and thermal characteristics of Titania nanofluids in a cylindrical annulus by considering ethylene glycol, engine oil, and water are used as base fluids. They capitalized Maxwell model for convective heat transfer in nanofluids and solved the constitutive equations by finite volume method [25]. Williamson fluid flow over a non-permeable stretching sheet by measuring the slope of the linear regression line of the data points for various involved parameters.

It is well known fact that most of industrial, physical and biological processes are complexly structured and mathematical modeling is considered as best tool for their interpretation. Over the years, various phenomena are narrated mathematically in the form of differential equations but like the intrinsic features of these procedures the attained differential equations are intricate in nature. So in recent years various analytical and numerical approaches are constructed to handle such valuable procedures. In this regard analytical methods by using fractional derivative has played significant role in handling and narration of such multifarious problems. Some recent investigations on finding solution of such problems by way of fractional derivatives are mentioned here. Atangana and Gomez [26] addressed the numerical approximation of fractional differentiation based on the Riemann-Liouville definition. Saad and Gomez [27] constructed solutions for the fractional cubic isothermal auto-catalytic chemical system with Caputo–Fabrizio and Atangana–Baumann fractional time derivatives in Liouville–Caputo sense. Delgado et al. [28] probed analytical solution of the time fractional diffusion equation and fractional convection-diffusion equation. Goufo et al. [29] performed numerical approximations of the solutions are presented and illustrated by graphical representations exhibiting a clear comparison between the dynamics under the influence of Mittag-Leffler law and those under the exponential law. A robust computational algorithm of homotopy asymptotic method for solving systems of fractional differential equation was constructed by Obidat and Sunil [30]. More literature about utilization of fractional derivatives is accessed through [31–37]. But in current work we are considering partial derivative of integral order so we are capitalizing finite element scheme for the computation of numerical results.

### Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>#EL</td>
<td>Number of Elements</td>
</tr>
<tr>
<td>#DOF</td>
<td>Number of degrees of freedom</td>
</tr>
<tr>
<td>p</td>
<td>Dimensionless pressure</td>
</tr>
<tr>
<td>ρ</td>
<td>Density of the fluid</td>
</tr>
<tr>
<td>ui</td>
<td>Dimensionless velocity components</td>
</tr>
<tr>
<td>γ</td>
<td>Shear rate</td>
</tr>
<tr>
<td>η_0</td>
<td>Viscosity at zero shear rate</td>
</tr>
<tr>
<td>U_{mean}</td>
<td>Mean inflow velocity</td>
</tr>
<tr>
<td>U_{max}</td>
<td>Maximum inflow velocity</td>
</tr>
<tr>
<td>n</td>
<td>Power law index</td>
</tr>
<tr>
<td>m</td>
<td>Consistency index</td>
</tr>
</tbody>
</table>
Fig. 1 – Schematic physical geometry.

The rationale of present effort is to disclose the features of power law material by constructing comparison with Newtonian fluid in a newly proposed physical configuration known as channel driven cavity. Solution of constructed differential system is sought by implementing finite element method. Variation in momentum distribution against different magnitude of Reynold number and power law index for Newtonian and non-Newtonian case is plotted.

Formation of stagnation points at different positions is interpreted via pressure plots against different magnitude of involved parameters. Line graphs are drawn to check the flow behavior near the cavity and at different positions of the channel to see the impact of non-linear viscosity.

Fig. 2 – Computational grid at level-1.

Fig. 3 – Sequence of grids on space mesh level: 1, 2, 3 (from left to right).

Fig. 4 – A $P_2 - P_1$ finite element pair-Location of degree of freedom.

Fig. 5 – (a–d) Velocity profile against Reynold number (Re).
2. Flow configuration and mathematical formulation

Consider a channel driven cavity shown schematically in Fig. 1. The cavity of unit dimension is adjusted on lower side of channel. The upper and lower walls of the channel are set at $u=v=0$ (no slip boundaries by setting). A parabolic inlet velocity with magnitude $U_{\text{max}}=0.3$ is injected to the channel and a do nothing boundary condition at the outlet is chosen. Power law fluid is considered representing the properties of shear thinning, shear thickening and Newtonian materials is considered.

The mathematical formulation of continuity and Navier-Stokes equations for incompressible Power law fluid is as under

$$\frac{\partial u_i}{\partial x_i} = 0. \quad (1)$$

Two cases for $\mu$ are considered i.e. for Newtonian fluid $\mu = \mu_0$ and $\mu(y) = m(y)^{n-1}$ for Power-law fluids. Here $m$ is the consistency index of the flow and $n$ is power law exponent responsible for shear thinning and thickening regimes.

3. Solution methodology

Mathematical modelling of most of engineering problems is highly complexed in nature so extraction of solution is also tough task. In this regards various analytical and exact approaches are available in literature [38–44] but each method has restrictions. Among these methods the most renowned
method is fractional order derivative which is the generalization of integral order derivative and obliged in many practical problems but in present communication the partial differential equation is generated in the form of integral derivative therefore we employed finite element scheme to narrate the flowing features of fluid in channel driven cavity. We apply Galerkin finite element method for the numerical approximation of the governing mathematical model. In this direction the conforming element pair $P_2 - P_1$ is selected for the velocity and pressure approximations. This element is a stable element pair satisfying inf-sup condition. The location of degrees of freedom for this finite element pair is shown in Fig. 2. This pair has 15 local degrees of freedom for two-dimensional flows. Newton’s method is applied to solve discrete non-linear algebraic systems and the inner linear sub problems are solved using a direct solver. The convergence criteria for the nonlinear iteration is set as

$$\left| \frac{x^{n+1} - x^n}{x^{n+1}} \right| < 10^{-6}$$

where $x$ represents the general solution component (Figs. 3 and 4).

To go through higher refinement levels, one element is converted into four elements of smaller size. The refinement procedure is explained in Fig. 2.

4. Results and discussions

4.1. Newtonian case

Variation in velocity profile against different magnitude of Reynold number (Re) ranging from $10 \leq Re \leq 100$ is adorned in Fig. 5(a–d). Since fluid is entering into the channel with parabolic profile so at the inlet and outlet the fluid enters and leaves with same parabolic summary and very negligible change in velocity occurs in the other regions on domain.

Depiction about velocity magnitude at different values of Reynold number (Re) through stream lines is sketched in Fig. 6(a–d). From the manipulation of stream lines it is seen that eye of vortex with in the cavity continuously deforms.

Fig. 8 – (a–d) Velocity profile against Reynold number (Re).

Fig. 9 – (a–d) Streamline with $n = 0.5$ for different Reynold number (Re).
with the increase of Reynold number (Re). It is also deduced that at lower magnitude of Reynold number the vortex shows uniform pattern in contrast to higher magnitude of (Re).

Fig. 7(a–d) is plotted to measure change in pressure against various values of Reynold number (Re). Since pressure is always linear in nature so with in the channel the position where no disturbance is felt by Newtonian fluid it show straight lines. Stagnation points at the corners of cavity are also formed where pressure at maximum intensity.

4.2. Non-Newtonian case (Power-law)

Change in velocity with different value of Reynold number (Re) for shear thinning case i.e. n = 0.5 is depicted in Fig. 8(a–d). As Reynold number (Re) signifies the ratio of inertial forces to viscous forces so by increasing the Reynold number (Re) viscosity decreases and hence flow rate increases.

Stream lines for shear thinning case (n = 0.5) and for different values of (Re) is expressed in Fig. 9(a–d). It is manifested that change in fluid occurs in the vortex pattern with in cavity because when Reynold number (Re) has lowest magnitude the viscosity effects are dominant so stream lines are symmetric in cavity. Whereas deformation in stream lines and eye of vortex appears at high value of Reynold number (Re) because inertial forces become dominant which effects in abrupt change in flow. Furthermore, it is also disclosed that slope of stream lines in channel is constant whereas in cavity slope of lines varies.

Effect of Reynold number (Re) on pressure profile is addressed in Fig. 10(a–d). It is concluded that pressure of fluid at inlet and at corner of cavity has optimum value and uniformity is generated near the outlet of channel. It is also interpreted that pressure has a stagnant value at the right upper corner of the cavity.

Effect of power law index (n) on velocity magnitude for shear thinning i.e. n = 0.5, Newtonian fluid i.e. n = 1 and shear thickening fluid i.e. n > 1 are divulged in Fig. 11(a–d). It is perceived that flow stream of fluid for shear thinning case is faster that the Newtonian fluid and shear thickening.
Fig. 12 – (a–d) Streamline variation against power law index (n).

Fig. 13 – (a–d) Pressure variation against power law index (n).

Fig. 14 – (a–d) Viscosity profile against power law index (n).
Table 1 – Mesh statistics at different refinement levels.

<table>
<thead>
<tr>
<th>Refinement level</th>
<th>Number of elements</th>
<th>Degree of freedom</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>520</td>
<td>2676</td>
</tr>
<tr>
<td>2</td>
<td>854</td>
<td>4346</td>
</tr>
<tr>
<td>3</td>
<td>1272</td>
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<td>4</td>
<td>2166</td>
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<td>5</td>
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<tr>
<td>6</td>
<td>5078</td>
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<td>60,403</td>
</tr>
<tr>
<td>8</td>
<td>31,462</td>
<td>148,003</td>
</tr>
<tr>
<td>9</td>
<td>44,464</td>
<td>206,498</td>
</tr>
</tbody>
</table>

Table 2 – Number of non-linear iterations for various n.

<table>
<thead>
<tr>
<th>n</th>
<th>No. non-linear iterations</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>9</td>
</tr>
<tr>
<td>0.8</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1.1</td>
<td>3</td>
</tr>
<tr>
<td>1.3</td>
<td>4</td>
</tr>
<tr>
<td>1.5</td>
<td>5</td>
</tr>
</tbody>
</table>

It is observed that velocity magnitude for shear thinning case is higher and lowest for shear thickening fluids.

Cutline at x = 1 and 0 ≤ y ≤ 2 is generated in Fig. 15 (c). From the display it is observed that initially velocity is zero due to the presence of cavity and fluid fells in to the cavity and after coming outward from cavity it will move with maximum pressure and retain parabolic profile.

Fig. 15(a–e) is presented to evaluate velocity distribution at different locations of channel driven cavity. All of these profiles validate the parabolic inlet flow. From all of above line graphs, it is disclosed that velocity magnitude for shear thinning case is more than the Newtonian and shear thickening cases. The reason behind this fact that in shear thinning fluids the viscosity of fluid decreases in contrast to shear thickening fluids (Table 1).

Non-linear iteration representing the non-linear behavior in velocity against power law fluid material parameter (n) is enumerated in Table 2. From the collected data it is concluded that for lower magnitude of (n) expresses the shear thinning case and flow rate raises in this case because the viscosity of fluid decreases. This decrease in viscosity generates effective non-linearity in fluid flow. It is also manifested that magnitude of non-linearity decrease for Newtonian case (n = 1)
and suddenly boosts up for shear thickening case. The abrupt fall down of non-linear iteration is due to fact that viscosity becomes constant in the Newtonian fluids.

5. Conclusions

The mathematical and numerical aspects of generalized Newtonian material filled in a channel driven cavity are addressed here. The flow field properties for shear-thinning fluids \( n < 1 \), Newtonian fluid \( n = 1 \) and shear-thickening fluid \( n > 1 \) are examined with the help of the finite element method. Stream lines formation, pressure contours and viscosity plots against different involved parameters are deliberated. From the inference of work following key findings are listed

i) Slopes of stream lines in channel are uniform whereas in cavity vortices are formed.

ii) Velocity magnitude for shear thinning case is excessive than the Newtonian and shear thickening case.

iii) More non-linear iterations are required as the flow regime deviated from Newtonian case to shear thinning and shear thickening regimes.

iv) Pressure has stagnant value at the right upper corner of cavity.

v) By increasing the Reynold number the velocity increases which is justified by stream line plots.

Conflict of interest

Authors do not have any conflict of interest.

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REFERENCES


