Original Article

Fatigue damage model of woven glass-epoxy fabric composite materials

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ABSTRACT

Based on the Stiffness degradation rule of woven glass fiber composite under fatigue loading, a phenomenological fatigue damage model containing two material parameters in which first is directly proportional to the fatigue life and second is inversely proportional to fatigue loading level is presented in this paper by analyzing damage development effects with very nice result as fatigue modulus is inversely proportional to fatigue strain. Experimental data from Tensile Fatigue Test were employed to verify the model, and the results show that the model can describe the damage evolution of woven glass-epoxy composite laminates under different fatigue loading by verifying the predicted fatigue life of composites. Degradation of Young's Modulus at different loading cycles, damage development corresponding to Normalized life, its accumulation subjected to constant amplitude fatigue loading including its verifications are well discussed.

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1. Introduction

Fatigue is defined as the dominant failure characteristic of composite structures under different cyclic loading which creates damage and degradation of material property [1]. The foundation to predict the fatigue life of composite structures, a damage evolution analysis is important for familiarity of the fatigue behaviour investigation required for design of composite engineering structures. Fiber breakage, matrix cracking and delamination is due to composite fatigue [2]. Hence, to predict the damage evolution as the complexity of the fatigue damage, the base line for the foundation of composite fatigue analysis is the modeling [3]. For tracking the alteration of material properties such as strength and stiffness, capability for simulating the damage is mainly through macroscopic models [4]. Regarding the degradation of initial fatigue life strength, several models corresponding to the residual strength are available in [5,6]. To assess the remaining life of structure, the optimum measurement of residual strength can be done by non-destructively [7], which is not suitable for tracking and predicting the fatigue damage. During the service life of the structure, for frequently or even continuously monitoring, non-destructive parameters as a form of stiffness is used [8,9].

In recent decades, several models based on stiffness regarding fatigue life prediction or fatigue damage evolution are presented in [10–19] which extended the measurement of Young's Modulus of the Materials. At the stage of final failure, most of above stated models are unable to predict the damage stated in [20]. Another main deficiency in most of these models are requirements of high numbers of parameters with realistic experimental data.

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For the limited range of loading level and for common fibers, Wu and Yao [21] had presented a stiffness-based damage evolution model of all stage with minimum requirements validating realistic in many of the similar models for specific types of composites and not evaluated in a wide range of fatigue loading levels. Another modified model validation for various fibers in a very limited range of loadings are described by authors [22]. Under non-constant amplitude loading, for the prediction of life of composites, rate-based stiffness degradation model has proposed by Peng et al. [23]. So, for composite structures, fatigue life prediction plays a vital importance role. To save the composite structure from catastrophic failures, performance of the structure in early life stage must be evaluated. Based on the residual strength or stiffness, some studies have been conducted. Assuming the association of cumulative damage with strength decrease, Yao and Himmel [24] had proposed a damage model in glass and carbon fiber reinforced plastics.

As known, complexity of fatigue in composites is more complexity than metals and matrix cracking, interfacial debonding, delamination and fiber breakage are basically four types of failure occurs. Based on different literature and a great deal of fatigue experimental investigations, a lot of linear and nonlinear damage models based on laboratory data [25–42], related to stiffness degradation, strength degradation and energy dissipation of composites, have been prescribed to describe the damage formulation of materials.

Fatigue Damage Model of woven glass fiber composite is developed based on the stiffness degradation rule. The evolution of model contains two material parameters. Based on the Stiffness degradation rule of woven glass fiber composite under fatigue loading, a phenomenological fatigue damage model containing two material parameters in which first is directly proportional to the fatigue life and second is inversely proportional to fatigue loading level is presented in this paper by analyzing damage development effects. Experimental data from Tensile Fatigue Testing were employed to verify the model, and the results show that the model can describe the damage evolution of woven glass-epoxy composite laminates under different fatigue loading by verifying the predicted fatigue life of composites.

### 2. Damage model

With the increment of the loading cycles, degradation of stiffness and elastic moduli regarding material properties at micro local field creates non-inverse changes under cyclic stress and strain causes fatigue failure of the composite.

Based on fatigue behaviour in the investigation regarding stiffness degradation creates damage development, the change in the Elastic Moduli can be find out briefly described by Reifsnider [34] and finally concluded non-linear characteristic of fatigue damage. At the beginning of the fatigue life, many more micro non-interactive cracks are developed in the form of Matrix, the fiber failure, interfacial debonding and delamination occurs when the matrix crack density reaches saturation after that the phenomenon of rapidly development of damage causes failure of material damage called “sudden death” occurs in the end of fatigue life period.

![Fig. 1 – Damage with respect to Cyclic Ratio.](image1)

![Fig. 2 – Rate of Damage w.r.t. Cyclic Ratio.](image2)

### Table 1 – Value of A and B.

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>A</th>
<th>B</th>
<th>A</th>
<th>B</th>
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<td>4.5</td>
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<td>5.5</td>
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<td>5.5</td>
<td>3.79</td>
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<td>1.5</td>
<td>1.445</td>
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<td>4.125</td>
<td>3.5</td>
<td>2.785</td>
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</tr>
<tr>
<td>2</td>
<td>1.78</td>
<td>6</td>
<td>4.46</td>
<td>4</td>
<td>3.12</td>
<td>8</td>
</tr>
</tbody>
</table>

Many more non-linear damage evolution models [42] were presented for optimization of time and cost of fatigue experiment. It is widely accepted that the model regarding stiffness degradation of woven glass-epoxy composite laminate by theoretically and experimentally by describing the initiation of damage at beginning or/and middle period of fatigue life but unable to describe the fatigue behaviour at the whole period of life related to fatigue as shown in Fig. 1. Along loading directions, to describe the stiffness degradation rule of composite mechanics, a versatile new fatigue damage model is described by Eq. (1),

\[
D(n) = \frac{E_0 - E(n)}{E_0 - E_f} = 1 - \left(\frac{n}{N}\right)^B A
\]
Where, $E_0$, $E_1$, $E(n)$ Young’s modulus at the initial, failure and material subjected to $n$th cycling loading respectively, $n$ and $N$ be the cycle and fatigue life, $A$ and $B$ are model parameters, $D(n)$ is fatigue damage equals to 0 when $n = 0$ and 1 when $n = N$.

\[ D = ABx^B \left( 1 - x^B \right)^{A-1} \]  

The rate of damage initiation in between the starting period and the ending period of fatigue life on the basis of damage evolution characteristic of composite materials, as shown in Eq. (3), as assumption is proposed relate to the rate at any normalized life $x_1$ and $x_2$ ($0 < x_1 < x_2 < 1$) which are same then the parameter $A$ and $B$ from Eq. (2) can be expressed as

\[ A = 1 + (B - 1) \frac{\lg x_1}{\lg (1 - x_1)} \]  

Verifying the relation between $A$ and $B$ in Eq. (3) mathematically gives the approximate linear relation,

When the $x_1$ and $x_2$ are discretionarily given. Therefore, the Eq. (3) can be approximately expressed as

\[ A = pB + q \]  

Where, $p$ and $q$ are constants. A quantitative relationship between the fitting parameters $A$ and $B$ for fitting the values in fatigue cyclic relation is proposed and shown as in Table 1,

\[ A = 0.678 + 0.44 \]  

When the composite laminate is subjected to the lesser ultimate strength, the ratio of stress or strain i.e. $R$, the fatigue life under the applied loading or bigger than applied loading is bigger fatigue damage than initial fatigue damage. The fatigue parameter $B$ in Eq. (1) is the indicator of fatigue laminate damage in the initial period of fatigue life. The lesser value of $B$ indicates for higher fatigue damage. Therefore, fatigue parameter $B$ is proportional to fatigue life $N$ and is inversely proportional to the fatigue loading level $\sigma_{max}/\sigma_{ult}$.

\[ B = k \frac{\lg N}{(1 - R) \left( \frac{\sigma_{max}}{\sigma_{min}} \right)} \]  

Where, $\sigma_{max}$ and $\sigma_{ult}$ is the maximum and ultimate strength and $k$ is a proportional constant. Damage and its rate of development is shown in Figs. 1 and 2.

4. Damage accumulation

When the constant amplitude of fatigue loading is subjected to the composite materials, then the initiation of material damage can be described by Eq. (1). Whatever, on the application of variable amplitude fatigue loading, the developed damage in the former stage loading will affect the quantity of produced damage in the next stage of loading. At that condition, the
accumulative fatigue damage $D(n)$ subjected to the ith loading of composite material is calculated as

$$D(n_i) = 1 - \left( 1 - \left( \frac{n_i + n_{i-1}}{N_{i-1}} \right)^{B_i} \right)^{A_i}$$  \hspace{1cm} (7a)

$$n_{i-1} = N_i \left( 1 - \left( 1 - \left( \frac{n_{i-1} + n_{i-2}}{N_{i-2}} \right)^{B_{i-1}} \right)^{A_{i-1}} \right)^{1/B_i}$$  \hspace{1cm} (7b)

Where, $A_i$ is the parameter under ith fatigue loadings, $A_{i-1}$ is the parameter under (i-1)th fatigue loading, $B_i$ and $B_{i-1}$ is the parameter under ith and (i-1)th fatigue loadings. Similarly, $n_i$ and $n_{i-1}$ are the ith and (i-1)th cyclic loading and $N_i$ and $N_{i-1}$ are the ith and (i-1)th applied fatigue loadings, whereas, $n_{i-1}$ indicates the equivalent fatigue cycles. Since, the equality in damage production, the equivalent cycles $n_{i-1}$ fatigue cycle is equal to sum of $(n_{i-1} + n_{i-2})$ under (i-1)th fatigue cyclic loading and is greater than or equals to 2, $n_{i-0} = 0$. The critical damage at the last step of fatigue cyclic loading is at Mth cyclic loading is defined as

$$D(n_M) = 1$$  \hspace{1cm} (8)

Fatigue Modulus

$$E = \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{\epsilon_{\text{max}} - \epsilon_{\text{min}}}$$  \hspace{1cm} (9)

Fatigue Modulus evolution model

$$E^n = E_0 \left( \frac{E_0}{E} + \frac{\alpha}{\beta} \left( \frac{\beta}{\beta - \frac{\alpha}{2D}} - 1 \right)^{\frac{\beta}{\alpha}} + 2D \right)$$  \hspace{1cm} (10)

Eq. (9) interprets the Young’s Modulus evolution model whereas Eq. (10) illustrates the fatigue strain evolution model corresponding to maximum and minimum stress to their normalized fatigue life.

5. Verification

According to the experimental data, it is a good regression that the proportional constant $k$ equals 0.06. The results show that the difference between the predicted residual fatigue life and the experimental data are acceptable because of the big scatter of fatigue life and most points are within 3 times range as shown in the Figs. 3 and 4. Experimental fatigue life among composite samples subject to the same fatigue loading is very different and the difference of experimental data is even bigger than 10 times. The predicted residual fatigue life by the proposed algorithm is in good agreement with the experiment, considering the bigger scatter of composites.

6. Conclusion

Based on the stiffness degradation rule of composite materials under fatigue loading, a phenomenological fatigue damage model was presented in this paper. The numerical examples show that the fatigue damage model is capable to describe the non-linear damage evolution in the whole fatigue life period of material. The characteristics of damage development of woven glass composite materials are analyzed and the quantitative relation is achieved in which the parameters of the model are proportional to the fatigue life and inverse proportional to the fatigue loading level. Also fatigue strain relates inverse to the fatigue modulus. The theoretical model is in agreement with the experimental results of residual fatigue life for woven glass composite materials.
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Conflicts of interest
The authors declare no conflict of interest.

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