Original Article

Activation energy on MHD flow of titanium alloy (Ti₆Al₄V) nanoparticle along with a cross flow and streamwise direction with binary chemical reaction and non-linear radiation: Dual Solutions

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Abstract

In this work, efforts are made to scrutinize the influence of MHD on nonlinear radiative flow comprising Ti₆Al₄V nanoparticle through streamwise direction along with a crossflow. Activation energy with binary chemical reaction is also explored through cross-flow comprising Ti₆Al₄V nanoparticle which has not been discussed yet. Similarity variables are employed to transform PDEs into nonlinear ODEs system. Then, it executed numerically through bvp4c from Matlab. Multiple solutions analysis is used to obtain first and second solutions. Influences of controlling non-dimensional parameters on liquid velocity and fluid temperature are scrutinized through the assist of diagrams. Results disclose that the drag surface force and rate of heat transfer increase for greater values of suction, while the rate of mass transfer decreases. Besides, multiple results are noticed for certain values of the moving parameter.

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1. Introduction

There are numerous substantial geometries that contract through the complex phenomena in line with the industrial requirement. The vicinity of flow from a surface is a vital part of aerodynamics and engineering. When a liquid moving through a sheet in aerodynamics. The layer of the boundary showing the curvature of the streamlines varies through the free stream. Since the gradient of pressure is at a 90-degree angle to the flow of the pathway of the free stream. The pressure gradient which is taken inside the layer of the boundary is not exaggerated because of the sheet distance. But various other elements work, for instant boundary surface, sharper bend when contrast to the flow of velocity of the free stream. Secondary or crosswise flow captures when the components of velocity along the boundary are perpendicular to the free-stream route. In such flows, the activity of transverse is considered to be completely expanded. Such flows can be
Nomenclature

B  variable magnetic field
B₀  strength magnetic field
Cᵢ, Cᵢ  skin friction coefficient in x- and z- directions
C  concentration of nanofluid
Cᵢ  concentration of nanofluid at wall
D  mass diffusivity of the base liquid
Dᵢ  mass diffusivity of the nanofluid
Eₐ  activation energy
f  dimensionless activation energy
k  thermal conductivity of the base liquid
kᵢ  thermal conductivity of nanofluid
kᵢ²  constant of chemical reaction rate
k*  coefficient of Rosseland mean spectral absorption
M  magnetic parameter
n  rate of a fitted constant
Nuₙ  Nusselt number
Pr  Prandtl number
qᵢ  radiative heat flux
qᵢₑ  heat flux
Rₑ  radiation parameter
Reₙ  local Reynolds number
Sc  Schmidt number
Shₙ  Sherwood number
T  temperature
Tᵢ  ambient temperature
Tₑ  fluid temperature at wall
U  constant velocity
u, v, w  velocity components
x, y, z  Cartesian coordinates

Greek symbols
αᵢ  thermal diffusivity of nanofluid [m²/s]
β  reaction rate
γ  concentration of nanoparticle
δ  temperature difference parameter
κ  Boltzmann constant
λ  dimensionless constant
μᵢ  thermal diffusivity of base fluid
μᵢₑ  thermal diffusivity of nanofluid
ϕ  volume fraction of nanoparticle
θ  dimensionless temperature
θₑ  ratio of temperature
ρᵢ  density of base liquid
ρᵢₑ  density of nanoliquid
(ρₑ)ᵢₑ  heat capacity of the nanofluid
τₑ  shear stress
ψ  stream function
η  similarity variable
σ*  Stefan-Boltzmann constant
σᵢ  electric conductivity of the base fluid
σᵢₑ  electric conductivity of the nanofluid

Superscripts
'  derivative w.r.t. η

Discovered in several circumstances in mechanical engineering, phenomena of wind flow and aerospace.

Jones [1] discussed the motivating outcomes for the problem of the crossflow. He showed the impact of sweepback on the separation and boundary layer. He also observed that there was a division of a swept wing or yawed because of the ratio of high aspect and thus few layers of boundaries and isolate the phenomenon are achieved through the velocity of crosswise. Sears [2] scrutinized the laminar flow through an immeasurable swept wing involving crossflow. He considered the gradient of pressure which is normal to the edge of the surface in the free-stream. Dwyer [3] introduced the innovative technique to obtain the solution of three-dimensional equations accurately. He utilized this technique to a problem that exhibits significant phenomena of the crossflow. New solutions were proposed by Weidman [4] which involve the boundary layers with cross-flow through a leading irregular edge. He discussed five distinct problems connected to cross-flow inside the theory of boundary layers and obtained novel solutions to transverse flow for stream-wise laminar flows. Karabulut and Ataer [5] investigated the steady-state flow with the characteristics of heat transfer from the cross-flow in a cylinder coordinate. They utilized finite difference techniques to gain the numerical solution for compressible as well as for incompressible flows. Three-dimensional bounded flows in the presence of span wise-cross free stream through the stirring boundary were examined by Fang and Lee [6]. Bhattacharyya and Pop [7] scrutinized the dissipating flow with characteristics of heat transfer in the direction of the cross-flow. The multiple nature of solutions were reported. Recently, Haq et al. [8] examined the influence of viscous-dissipation on the viscous liquid with heat as well as the mass transfer of the moving heated surface along the direction of stream-wise with the cross-flow.

A metallic liquid or a solid which can be compiled through non-homogenous or homogenous mixtures of two or more metalloid nano-meter sized elements is known as an alloy. Nowadays, the alloy can be utilized extensively for assigning the precise mixtures of physical characteristics. A few examples of alloys are the solder, gold, phosphor bronze, steel, and brass. These have an extensive range of applications for example technology of aerospace science, replacement process of the hip joint, implantation of surgical, technology of advanced powder and numerous biological treatments. Commonly, alloys are applied in handling systems of fabrication, hot and cold rolling sheets construction and many more. More about alloys applications are presented in articles [9–12]. In recent times, Raju et al. [13] discussed the influence of heat sink/source on MHD flow from cone comprising water and kerosene-based titanium alloy nanomaterials with temperature-dependent viscosity and viscous-dissipation. They scrutinized that the coefficient of friction factors has a smaller value in water (Ti₆Al₄V) titanium alloy compared to kerosene titanium alloy. Impact of iso-flux heat source on natural convective flow in a cylindrical annulus with stability of heat transfer was discussed by Mebarek-oudina [14]. He showed that the flow control and rate of heat transfer can be controlled by varying the heat source length. Shashikumar et al. [15] argued the impact of entropy generation on MHD nanofluid comprising titanium.
alloy and aluminum nanoparticles through micro-channels in the presence of convection conditions and partial slips. Recently, Souyeh et al. [16] scrutinized the behavior of slip effect on radiative flow containing ferromagnetic as well as titanium alloy nanomaterials suspended in a dusty liquid. They explored that the hybrid nanomaterials played a role more effectively in the process of heat transportation compared to base nano liquid. The free convective flow of water b/w two co-axial cylinders with constant heat source was discussed by Gourari et al. [17]. Mebarek-oudina and Bessaih [18] explored the impact of two heat sources of distinct lengths on natural convective flow containing water based nanoparticle from a vertical cylindrical annulus. They obtained the numerical solution by utilizing finite volume technique. Raza et al. [19] examined the radiation impact of MHD flow in a channel comprising molybdenum disulfide nanoliquid with varying walls and shapes. Mebarek-oudina [20] scrutinized the thermal and hydrodynamic characteristics involving Titania nanoparticle of three distinct regular liquids namely, oil, water and ethylene glycol from a cylindrical annulus in the presence of heat source. Recently, Raza et al. [21] explored the influences of slips on MHD flow containing nanoliquid with Williamson fluid from a stretched surface.

In 1889, Swante Arrhenius was introduced the idea of activation energy. Activation energy is the precise amount of minimum energy that is needed through the reactants for the occurrence of a chemical effect. It may be taken as the height of potential barriers or energy which splits both of the minima of the products and reactants of potential energy. This phenomenon has copious appliances in the field of chemical engineering, oil reservoirs or geothermal engineering, mechanics of base fluid and oil emulsion and processing of food. The pioneering work of Bestman [22] explained the behavior of activation energy with the binary reaction in natural convective flow embedded in a porous medium. Maleque [23] investigated the impact of MHD on time-dependent free convection flow from a flat sheet with activation-energy and binary-reaction. Abbas et al. [24] examined the impact of thermal radiation on an unsteady flow involving Casson liquid from the stretched/shrinking surface with activation energy and binary chemical reaction. The influence of activation energy on the MHD flow of Carreau nanoliquid with mixed convection and viscous dissipation was examined by Hsiao [25]. He used an innovative parameters control technique to endorse the efficiency of economic manufacturing through energy extrusion. Zaib et al. [26] scrutinized the effects of activation-energy and binary-chemical reaction on nonlinear radiative flow containing Casson nanoliquid through a plate embedded in the permeable medium. Recently, Javed et al. [27] conferred the impact of MHD on radiative stagnation-point flow with activation-energy and binary-reaction with erratic variable thickness.

A review of the literature reveals that no one yet discussed this type of model. Therefore, we are examining the impact of activation energy on cross-flow with streamwise direction involving titanium alloy nanoparticle with nonlinear radiation. This investigates added a novel approach for scientists and researchers to find out the nanofluid characteristics. The resultant model is solved numerically via bvp4c. The influences of the substantial constraints are argued through the assist of graphs and tables.

2. Problem Formulation

Problem formulation is based on cross-flow and streamwise within the approach of the boundary layer. Based on the phenomena of cross-flow, we examine the three-dimensional flow containing TiAlV alloy nanoparticle from a heated surface as shown in Fig. 1. Also, the influences of nonlinear thermal radiation along activation-energy and binary-chemical reactions are invoked. The surface is moving through the constant uniform velocity $-\lambda U$ placed at $x$ equal to zero that is out of the origin, where $x$ is taken along the surface, $\lambda$ and $U$ are the dimensionless constant and constant velocity, respectively. The variable magnetic field acts as $B = B_0(2x)^{-\alpha}$ and is applied normal to the surface. Besides, $T_\infty$ and $C_\infty$ are the constant temperature and concentration of fluid, while $T_\infty$ and $C_\infty$ are the ambient temperature and concentration, respectively. Moreover, it is perceived that the secondary flow or cross-flow has a wide range of extent in the direction of span-wise & therefore, fully extended. Thus, velocity, temperature, and concentration fields are free from the $z$ coordinate. The constitutive equations in terms of PDE’s via the approximation of the boundary layer scaling are [7,8,28]

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \frac{\mu_{nf}}{\rho_{nf}} \left( - \frac{\partial^2 u}{\partial y^2} \right) = \frac{\sigma_{nf} B^2}{\rho_{nf}} (U - u), \tag{2}
\]

\[
u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + \frac{\mu_{nf}}{\rho_{nf}} \left( - \frac{\partial^2 w}{\partial y^2} \right) = \frac{\sigma_{nf} B^2}{\rho_{nf}} (w_0 - w), \tag{3}
\]

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} - \frac{\alpha_{nf}}{\rho_{nf}} \frac{\partial^2 T}{\partial y^2} = - \frac{1}{\rho_{nf} c_{nf}} \frac{\partial q_i}{\partial y}, \tag{4}
\]

\[
u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_{nf} \frac{\partial^2 C}{\partial y^2} - k_f \left( \frac{T}{T_\infty} \right)^n e^{-\frac{E_a}{RT}} (C - C_\infty). \tag{5}
\]
along boundary conditions are
\begin{align}
  u + \lambda U &= 0, \quad v = u_0, \quad w = 0, \quad T - T_w = 0, \quad C - C_w = 0 \text{ at } y = 0, \\
  u - U &= 0, \quad w = 0, \quad T - T_\infty = 0, \quad C - C_\infty = 0 \text{ as } y \to \infty.
\end{align}
(6)

where the components of velocity are \((u, v, w)\) that run along \(x-, y-,\) and \(z-\) axes, respectively, \(T\) and \(C\) are the temperature and concentration of nanofluid respectively, \(k(T/T_\infty)^n \exp (-E_a/kT)\) the Arrhenius function with \(k\) Boltzmann constant, \(k^2\) constant of chemical reaction, and \(n\) rate of a fitted constant that lies \(b/w - 1 < n < 1\).

Thermophysical properties are described as [28]
\[(\alpha p)_f = \phi (\alpha p)_f \quad (\phi - 1) \quad (\alpha p)_f, \quad \alpha_{nf}(\alpha p)_f = k_{nf},\]
\[
\mu_{nf}(1 - \phi)^{2.5} = \mu_f, \quad k_{nf} = \left(\frac{k_s + 2k_f}{k_s + 2k_f} + 2\phi \left(\frac{k_s - k_f}{k_s - k_f}\right) \mu_{nf}\right) = \phi \rho_s - (\phi - 1) \rho_f.
\]
(7)

where \(\phi\) stands for the volume fraction of nanoparticle, \((\alpha p)_f\) heat effective capacity, \(\alpha_{nf}\) thermal diffusivity, \(\mu_{nf}\) thermal conductivity with \(k_s\) designates thermal conductivity of nanoliquid and \(k_f\) thermal conductivity of the base liquid, \(\rho_f\) density of nanoliquid with the \(\rho_s\) density of nanoliquid, \(\rho_f\) density of base liquid, \(\sigma_{nf}\) electric conductivity of nanoliquid with \(\sigma_f\) electric conductivity of nanofluid and \(\sigma_f\) electric conductivity of base fluid, \(D_{nf}\) mass diffusivity of the nanoliquid and \(D_f\) mass diffusivity of the base liquid.

We implement the approximation of Rosslend diffusion [29] for \(q_r\) (radiative heat flux) as we presume that the boundary layer is optically thick and is expressed as
\[
q_r + 16\alpha^* \frac{n^2}{3k^*} \frac{\partial T}{\partial y} = 0.
\]
(8)

where \(\alpha^*\) and \(k^*\) designate for Stefan-Boltzmann constant and coefficient of Rosseland mean spectral absorption, respectively.

By defining the similarity transformation [7,8]:
\[
\eta = \sqrt{\frac{U}{2xU}}, \quad \psi = \sqrt{2xU/C} f(\eta), \quad u = Uf(\eta), \quad w = \omega g(\eta).
\]
\[
\psi = \sqrt{\frac{U}{2x}} (f(\eta) - \eta f'(\eta)), \quad \theta(\eta) = \frac{T - T_w}{T_\infty - T_w}, \quad \chi(\eta) = \frac{C - C_w}{C_\infty - C_w}.
\]
(9)

We assign the non-dimensional temperature as with \(T - T_w (1 + (\theta_w - 1)) = 0, \theta_w = (T_w/T_\infty) > 1\) called the ratio of temperature.

Putting Eq. (9) into (2)-(5) and using the expressions (7) and (8). The governing ordinary differential equations become as:
\[
\frac{f''}{(1 - \phi)^{3.5}} + \left(1 - \phi + \frac{\phi 0_k}{\rho_f} \right) f'' + M \left(1 - \frac{3}{(\phi - \phi_f)} \phi \right) \left(1 - g\right) = 0.
\]
(10)

\[
\frac{g''}{(1 - \phi)^{2.5}} + \left(1 - \phi + \frac{\phi 0_k}{\rho_f} \right) g'' + M \left(1 - \frac{3}{(\phi - \phi_f)} \phi \right) \left(1 - g\right) = 0.
\]
(11)

\[
\frac{\eta''}{3R_d \left(\frac{k_s + 2k_f}{k_s + 2k_f} + 2\phi \left(\frac{k_s - k_f}{k_s - k_f}\right) \mu_{nf}\right)^2} + 4\phi (\theta_w - 1 + 1)^2 \left(\phi - 1\right) \left(\phi - 1\right) + 12 (\theta_w - 1) (\theta_w - 1) + 1)^2 \left(\phi - 1\right)^2 = 0.
\]
(12)

\[
(1 - \phi)\chi'' + Sc \eta f' - \beta S c (1 + \delta \phi)^n \exp \left[-\frac{E}{1 + \delta \phi}\right] \chi = 0.
\]
(13)

The converted subject conditions are
\[
f(0) - S = 0, \quad f'(0) - \lambda = 0, \quad g(0) = 0, \quad \theta(0) - 1 = 0, \quad \chi(0) - 1 = 0.
\]
(14)

In above equations, \(R_d\) radiation parameter, \(M\) magnetic parameter, \(Pr\) Prandtl number, activation parameter \(E,\) reaction rate \(\beta,\) temperature difference parameter \(\delta\) are defined as
\[
R_d = \frac{k_s k_f}{4\sigma^* T_\infty^4}, \quad M = \frac{\sigma_f B_0^2}{\rho_f U}, \quad R_x = \frac{xU}{U}, \quad Pr = \frac{\eta U}{\eta_f}, \quad E = \frac{E_a}{kT_\infty}.
\]
\[
\beta = 2 \frac{R_p U}{U}, \quad \delta = \frac{T_w - T_\infty}{T_\infty}.
\]

2.1. Skin friction

Coefficients of skin friction over the stream-wise \(C_{fs}\) in the \(x-\) direction and cross-flow \(C_{fz}\) in the \(z-\) direction are written as [30]
\[
C_{fs} = \frac{M \eta_{nf}}{\rho_f U^2} f''(0) = \frac{f''(0)}{\sqrt{2\eta_{nf}}} = \frac{f''(0)}{\sqrt{2\eta_{nf}}}.
\]
\[
C_{fz} = \frac{M \eta_{nf}}{\rho_f W_0^2} g''(0) = \frac{g''(0)}{\sqrt{2\eta_{nf}}} = \frac{g''(0)}{\sqrt{2\eta_{nf}}}.
\]
2.2. Nusselt number

The local Nusselt number in dimensionless form is written as [30]

\[ Nu_x = -\frac{x \left( \frac{-k_j}{\eta_j} \right)_{y=0}}{\sqrt{2Re_s}} \]}

\[ \frac{k_j}{k_f} \left( T_w - T_{\infty} \right) - \frac{\left( k_j/k_f \right) \phi'(0)}{\sqrt{2Re_s}}. \]

2.3. Sherwood number

The local Sherwood number in dimensionless form is written as [28]

\[ Sh_x = \frac{S}{D_f (C_w - C_{\infty})} - \frac{\phi'(0)}{(1 - \phi) \sqrt{2Re_s}}. \]

where \( Re_x = xU/\nu \) called Reynolds number.

3. Results and Discussion

The nonlinear problem in terms of Partial differential equations (PDE) Eqs. [2–5] is transformed into nonlinear Ordinary differential equations (ODE) Eqs. [10–13] with pertinent restrictions Eq. [14] is unraveled by using bvp4c package from Matlab. For computational purposes, the fixed parameters throughout the study are considered as the following \( S = 1, M = 0.1, \delta = \beta = \varepsilon = \delta = \omega = 0.5, n = 0.4, \phi = 0.01 \) and \( \lambda = 0.1 \). The (Pr) Prandtl number for the water is set as \( Pr = 6.2 \) and the nanoparticles volume fraction is lying in the range of \( 0 \leq \phi < 1 \) in which the regular viscous fluid is occurred at \( \phi = 0 \) and had studied by Oztöp and Abu-Nada [31]. The numerical analysis for obtaining dual solutions is performed to study the MHD effect on a nonlinear radiative nanofluid flow such that the nanofluid is made of Ti₆Al₄V nanoparticles and water. Activation energy with binary chemical reaction is also explored through the stream or secondary wise and cross flow. The dual results are obtained and plotted in different graphs. During the numerical simulations, the thermophysical characteristics of base fluid water and nanoparticle Ti₆Al₄V are displayed in Table 1.

![Fig. 2 – Impact of S on (2Re_x)^{1/2}C_f.](image1)

![Fig. 3 – Impact of S on (2Re_x)^{1/2}C_f.](image2)

![Fig. 4 – Impact of S on (2Re_x)^{1/2}Nu_x.](image3)

Table 1 – Thermophysical properties of base fluid and Ti₆Al₄V.

<table>
<thead>
<tr>
<th>Material</th>
<th>Water</th>
<th>Ti₆Al₄V</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_p ) (J/kgK)</td>
<td>4179</td>
<td>0.56</td>
</tr>
<tr>
<td>( \rho ) (kg/m³)</td>
<td>997.1</td>
<td>4420</td>
</tr>
<tr>
<td>( k ) (W/mK)</td>
<td>0.613</td>
<td>7.2</td>
</tr>
<tr>
<td>( \sigma ) (S/m)</td>
<td>0.005</td>
<td>( 5.8 \times 10^5 )</td>
</tr>
<tr>
<td>( Pr )</td>
<td>6.2</td>
<td>–</td>
</tr>
</tbody>
</table>
detected from Fig. 2, that both solutions (first solution and as well as second solution) boost up with increasing $S$. It is also worthy to note that in the stretching case ($\lambda > 0$) the coefficient of skin-friction is larger than that observed for the shrinking case ($\lambda < 0$). A similar behavior (like Fig. 2) is observed in Fig. 3 which is plotted for the skin friction coefficient along $z$-axis $(2\text{Re}_x)^{1/2}\text{Cf}_z$ against $\lambda$ (stretching parameter/shrinking parameter) for different values of $S$. However, the variation in the first solutions is not as prominent as in Fig. 2. Note that the subscripts $x$ or $z$ in the local skin friction coefficient show its direction along $x$-axis or $z$-axis. In Fig. 3 different from Fig. 2, it is perceived that the coefficient of skin-friction in $z$-direction shows almost identical values for both the phenomenon of stretching and shrinking velocities especially for the solution of the first branch. Indeed, in the second branch, this variation is noticeable for the stretching and shrinking cases.

The heat transfer rate is computed in terms of the $(2\text{Re}_x)^{1/2}\text{Nu}_x$ and the rate of mass transfer is computed in terms of the $(2\text{Re}_x)^{1/2}\text{Sh}_x$ against $\lambda$ for enormous values of $S$ as shown in Fig. 4 and Fig. 5, respectively. More exactly, Fig. 4 shows a decreasing behavior of $(2\text{Re}_x)^{1/2}\text{Nu}_x$ for growing values of $S$ for both types of solutions (first solution and second solution). However, this behavior is opposite in Fig. 5, i.e. for decreasing values of the suction parameter $S$, $(2\text{Re}_x)^{1/2}\text{Sh}_x$ decreases. In the arguments of local Nusselt number and local Sherwood number, the situation in terms of stretching & shrinking velocities is totally different compared to Fig. 2 & 3, i.e. the local heat transfer rate as well as the local mass transfer rate is maximum for the case of shrinking velocity than that examined for the case of stretching velocity.

The consequence of the magnetic parameter $M$ against $\lambda$ where the critical values are $(\lambda_c = 0.32330, 0.44790, 0.58170)$ for the local co-efficient of skin-friction along $x$- and $z$- directions is highlighted in Figs. 6 and 7. Both cases of stretching and shrinking velocities are considered. Fig. 6 elucidates that with increasing magnetic parameter, the local skin-friction coefficient grows for both types of solutions (first and second solution). Furthermore, it is observed that $(2\text{Re}_x)^{1/2}\text{Cf}_x$ the stretching case is higher than the shrinking case. The behavior in Fig. 7 is quite different, here the $(2\text{Re}_x)^{1/2}\text{Cf}_z$ declines with growing the magnetic parameter $M$ values. The variation in the second solution is quite prominent compared to the first solution.

The effect of $M$ on the $(2\text{Re}_x)^{1/2}\text{Nu}_x$ and $(2\text{Re}_x)^{1/2}\text{Sh}_x$ along the stretching/shrinking parameter is studied in Figs. 8 and 9, respectively. The critical values against $\lambda$ for the enormous values of the magnetic parameter are the following $(\lambda_c = 0.32330, 0.44790, 0.58170)$ for both the rate of heat and mass transfer respectively. One can see the opposite trend of these dual solutions in both of these graphs. More exactly, for larger values of $M$, the first solution displays a down movement whereas for the second solution uptrend is noticed. Furthermore, these graphs illustrated that the local heat and mass transfer rates are quite smaller for the first solution as compared to the second solution.

$(2\text{Re}_x)^{1/2}\text{Nu}_x$ along $\lambda$ with the changed values of the radiation parameter $R_d$ is shown in Fig. 10. It is found that an
increase in $R_d$ causes a decreasing behavior in $(2\text{Re}_x)^{1/2}\text{Nu}_x$ for both solutions (first solution and second solution).

The effect of the ratio of temperature $\theta_w = (T_w/T_\infty) > 1$ along $\lambda$ on the $(2\text{Re}_x)^{1/2}\text{Nu}_x$ is examined in Fig. 11. It is originated that with increasing $\theta_w$, the rate of local heat transfer $(2\text{Re}_x)^{1/2}\text{Nu}_x$ decreases. The next two graphs (Figs. 12 and 13) are plotted for showing variations of activation parameter $E$ and temperature difference parameter $\delta$ for $(2\text{Re}_x)^{1/2}\text{Sh}_x$ both cases of stretching and shrinking velocities. An identi-
cal behavior is observed in both of these graphs. However, the mass transfer rate in Fig. 13 is higher as compare to Fig. 12.

Dual solutions of $f'(\eta)$ for three different values of suction parameter are plotted in Fig. 14. It is found that the first solution rises with growing values of $S$, whereas the second solution decreases for larger values of $S$. Fig. 15, presented the behavior of $f'(\eta)$ against $\eta$ for different values of $M$, an opposite trend is noted as compared to Fig. 14. In the first solution, velocity decreases with increasing $M$ whereas in the second solution $f'(\eta)$ increases for larger values of $M$. In physical sense, a similar behavior was expected as we know that a boost up the magnetic parameter generates more Lorentz forces which retard the fluid motion significantly and hence the velocity profile and the momentum boundary layer thickness decrease with increasing the magnetic parameter.

The effect of the suction parameter $S$ on the $f'(\eta)$ profile in z-direction $g(\eta)$ is shown in Fig. 16. It is initiated that the first solution upsurges for larger values of $S$ however, this trend is vice versa for the solution of the second branch i.e. with increasing suction parameter, the second solution decreases. The next figure (Fig. 17) indicates that $g(\eta)$ shows a falling behavior for the first solution for growing values of the magnetic parameter whereas $g(\eta)$ for the second branch increases for larger change choice of the magnetic parameter.

Figs. 18 and 19 are plotted in a sequence to expose the behavior of temperature profile $\theta(\eta)$ for various values of $S$, the
suction parameter and M magnetic parameter, respectively. More exactly, it is examined from Fig. 18 that increasing S, the temperature growth for the first solution, however, this trend is opposite for the second solution that temperature profile decreases with increasing S. Fig. 19, on the other hand, demonstrates that temperature profile increases for the first branch with an increasing magnetic parameter whereas decreases the second branch as the magnetic parameter is increased.

Figs. 20 and 21 are prepared in order to highlight the reassurance of S and M on the concentration profile of nanofluid. The aforementioned from Fig. 20 that the dual solutions show a declining behavior with growing values of S. Fig. 21 indicates that the impact of the magnetic parameter on the concentration profile of nanofluid is opposite for the upper branch compares to the lower branch results. More exactly, the first solution of the concentration field boosts up with the growing effect of the magnetic parameter whereas the second solution of the concentration field decreases with increasing magnetic parameter.

Figs. 22 and 23 are prepared to show the effect of δ and Sc on $\chi(\eta)$, respectively. Further, observed in Fig. 22 that in the first solution nanofluid concentration enhances with increasing δ,
however, this nanofluid concentration decreases with increasing $\phi$. Fig. 23 shows that the nanofluid concentration decreases for the dual solutions when the Schmidt number increases. Fig. 24 depicts the effects of $\beta$ on $\chi(\eta)$. A decreasing behavior is shown by nanofluid concentration $\beta$ on $\chi(\eta)$ for both the first and second solutions. More exactly, it is noted that the nanofluid concentration reduces when $\beta$ is increased. The nanoparticle volume fraction $\phi$ is depicted in Figs. 25, 26 and 28 that both branches’ solutions for the velocity and concentration were decreases due to increase the value of $\phi$. While an opposite behavior is observed against the temperature distribution which is shown in Fig. 27.

4. Conclusions

In this present work, we have incorporated the 3D boundary layer flow of nanofluid along with the stream or secondary wise and cross-flow directions. We have discussed more and analyzed the rate of heat transfer and mass transfer difference in the base fluid (water) by combining the titanium alloy nanoparticle (Ti$_6$Al$_4$V). Further, the activation energy, binary

Fig. 24 – Impact of $\beta$ on $\chi(\eta)$.

Fig. 26 – Impact of $\phi$ on $g(\eta)$.

Fig. 25 – Impact of $\phi$ on $f'(\eta)$.

Fig. 27 – Impact of $\phi$ on $\theta(\eta)$.

Fig. 28 – Impact of $\phi$ on $\chi(\eta)$. 

Fig. 23
chemical reaction and nonlinear thermal radiation impacts are also taken along x- and z- directions. The significant conclusions developed from this research are as pursues:

- The velocity and concentration along the x- and z- directions of the streamwise and cross-flow decline with growing \( \phi \). However, the temperature distribution increases due to \( \phi \).
- The velocity behavior is boosted up in the first solution when the suction parameter enhances and the reverse trend is noted for the second solution along the x- and z- directions.
- Further, the temperature and concentration boundary layers become thinner and thinner in both solutions due to S.
- The value of the magnetic parameter enhances, the first solution moves down and the second solution goes up in case of velocity, whilst the opposite trend is shown for the skin friction.
- Both solutions (first and second) for the temperature and concentration rise and decrease due to boost up the enormous values of the magnetic parameter, respectively.
- The temperature difference parameter raises against the concentration profile, the first solution boosts up while the second solution behavior subdue.
- Both the solutions decrease for the concentration profile against the growing values of the reaction rate and Schmidt parameter.
- Due to \( \eta_{0} \) and \( R_{D} \), decreasing behavior is observed for \((2Re_{k})^{1/2}Nu_{c}\), while the behavior of \((2Re_{k})^{1/2}Sh_{x}\) increases due to \( E \) and \( \delta \).

**Conflict of Interest**

Authors do not have any conflict of interest.

**REFERENCES**


