Original Article

Investigation on the material flow and deformation behavior during ultrasonic-assisted incremental forming of straight grooves

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A R T I C L E   I N F O

Article history:
Received 11 July 2019
Accepted 28 October 2019
Available online 14 November 2019

Keywords:
Incremental sheet forming
Ultrasonic-assisted straight groove forming
Material flow
Forming force
Acoustic softening

A B S T R A C T

The introduction of ultrasonic vibration is proved to be beneficial for the reduced force and improved formability for sheet forming processes. However, the vibration-induced material behavior is still not satisfactorily explained. The objective of this work is to investigate the effect of the ultrasonic vibration on the material flow characteristics and the plastic deformation mechanism during the incremental sheet forming of the straight groove. Both analytical and finite element models are established to explore the impacts of ultrasonic amplitude, frequency, step-down size and feed rate on material flow characteristics. It was found that the material flow area was dramatically increased after applying the ultrasonic vibration, and the influence of ultrasonic amplitude on material flow area is greater than that of frequency, which is similar to the effect of acoustic softening. In addition, the separation effect under different amplitudes and frequencies was analyzed through finite element modeling to partly explain the reduction of forming forces. The duty cycle was decreased as the amplitude increases as well as the frequency. Furthermore, the material flow behavior and the force reduction rate for different tool paths are explored. Finally, experiments were performed which further verified the reliability of the analytical and FE models. These findings provide a theoretical basis for further investigation of the mechanisms of ultrasonic-assisted incremental sheet forming process.

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1. Introduction

Incremental sheet forming (ISF) is a rising forming technology suitable for complex three-dimensional shapes, and that the application of ISF for rapid prototyping and small quantity production has great potential. Since Leszak [1] proposed the
Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_m )</td>
<td>ultrasonic amplitude</td>
</tr>
<tr>
<td>( A_{ui}(\gamma) )</td>
<td>the total material flow area at the section angle ( \gamma ) during UISF process</td>
</tr>
<tr>
<td>( A_{uiB}(\gamma) )</td>
<td>the overlapping area at the section angle ( \gamma ) between two successive cycles</td>
</tr>
<tr>
<td>( A_{uiE}(\gamma) )</td>
<td>the extra material flow area at the section angle ( \gamma ) induced by ultrasonic vibration</td>
</tr>
<tr>
<td>( A_{uiL}(\gamma) )</td>
<td>the material flow area at the section angle ( \gamma ) not induced by ultrasonic vibration during UISF process</td>
</tr>
<tr>
<td>( A_{ui2}(\gamma) )</td>
<td>the distinct material flow area at the section angle ( \gamma ) induced by ultrasonic vibration</td>
</tr>
<tr>
<td>( A_{ui3}(\gamma) )</td>
<td>the material flow area at the section angle ( \gamma ) without ultrasonic vibration</td>
</tr>
<tr>
<td>( f )</td>
<td>ultrasonic frequency</td>
</tr>
<tr>
<td>( h_p )</td>
<td>the actual depth between the formed surface and the unformed surface during the straight groove forming</td>
</tr>
<tr>
<td>( h_\gamma )</td>
<td>the actual processing depth at the section angle ( \gamma )</td>
</tr>
<tr>
<td>( h_1 )</td>
<td>the vertical distance between point ( A_1 ) and ( B_1 ) in one vibration cycle</td>
</tr>
<tr>
<td>( h_A )</td>
<td>the vertical distance between two forming steps at point ( A )</td>
</tr>
<tr>
<td>( L_{AB} )</td>
<td>the horizontal distance from point ( A_1 ) to point ( B_1 )</td>
</tr>
<tr>
<td>( L_3 )</td>
<td>the length of the straight groove</td>
</tr>
<tr>
<td>( R )</td>
<td>the radius of the forming tool</td>
</tr>
<tr>
<td>( T )</td>
<td>the period of each vibration cycle</td>
</tr>
<tr>
<td>( V )</td>
<td>the feed rate</td>
</tr>
<tr>
<td>( V_{ui} )</td>
<td>the total material flow volume during UISF process</td>
</tr>
<tr>
<td>( V_{uiB} )</td>
<td>the overlapping volume between two successive cycles</td>
</tr>
<tr>
<td>( V_{uiE} )</td>
<td>the extra material flow volume induced by ultrasonic vibration</td>
</tr>
<tr>
<td>( V_{ui1} )</td>
<td>the material flow volume not induced by ultrasonic vibration during UISF process</td>
</tr>
<tr>
<td>( V_{ui2} )</td>
<td>the distinct material flow volume induced by ultrasonic vibration</td>
</tr>
<tr>
<td>( X_{EB} )</td>
<td>the horizontal distance from point ( E_1 ) to point ( B_1 )</td>
</tr>
<tr>
<td>( Y_{EB} )</td>
<td>the vertical distance from point ( E_1 ) to point ( B_1 )</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>half of the contact angle</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>the angle corresponding to the position of any section</td>
</tr>
<tr>
<td>( \Delta z )</td>
<td>the step-down size</td>
</tr>
<tr>
<td>( \theta )</td>
<td>the inclined angle of the zigzag-type tool path</td>
</tr>
</tbody>
</table>

Throughout-thickness shear [2], the bending under tension [3], and the ‘noodle’ theory [4] have been proposed for the explanation of the enhanced formability in ISF. From the doctor’s thesis of Liu [5], the relation between the deformation mechanism and the process formability is depicted in detailed. Otherwise, some researchers focused on local deformation. Emmens and Boogaard [6] investigated the local deformation characteristics of the ISF process and claimed that the local deformation is an essential feature of ISF. Ma et al. [7] examined the mechanism of ISF in different regions and concluded that more materials are involved in the larger plastic deformation area. Bansal et al. [8] found that the proper deformation of materials during ISF is significant to avoid the occurrence of forming failure. Therefore, the material flow characteristic directly affects the plastic deformation mechanism during the process. Sznyler et al. [9] investigated the effects of process parameters on the material flow in ISF based on the developed concurrent multiscale numerical model. It was found that the strain and hardness distribution accumulate uniformly along the sample surface during the incremental forming.

Different strategies have been adopted to control the material flow behavior in ISF. Multi-stage single-point forming was proposed by Dufiou et al. [10] and adopted by Zhang et al. [11] to improve the material flow behavior and thickness distribution of the formed parts. Malhotra et al. [12] and Xu et al. [13] established analytical equations to analyze the material flow path in the cross-section and efficiently predicted the formation of the stepped feature in the multi-stage single-point incremental sheet forming process. Liu et al. [14] studied different strategies for the multi-stage single-point sheet forming process to control the material flow to avoid forming failure. The material flow can also be effectively controlled by the two-point forming strategy. Kim and Yang [15] pointed out that the distribution of the thickness becomes more uniform by the double-pass forming method for a designed shape. Liu et al. [16] investigated the influence of ramp angle, travel direction and tool path starting point on material thinning in the multi-stage two-point incremental sheet forming process. It was claimed that the inward material flow is the reason why the multi-stage two-point forming process showed better formability compared with single-stage two-point forming strategy. The alternating of tool path strategies is another effective method to control the material flow in ISF. Hirt et al. [17] proposed a tool path correcting method based on experiments to achieve a specified geometric tolerance. Malhotra et al. [18] proposed a novel toolpath strategy for double side incremental sheet forming process (DSIF) to stabilize the deformation into the local region around the contact point. Both the experimental and simulation results confirmed that this strategy could improve geometry effectively. At present, the investigation of the material flow area is limited in improving the thickness distribution by using different tool path strategies and multi-stage forming in the incremental forming process. However, there is a lack of analytical models for investigating the local material flow characteristic during ISF.

In recent years, it is noticed that ultrasonic vibration has been considered as an auxiliary energy to improve the performance in the typical forming processes. Blaha and Langenecker [19] first proposed the theory of volume effect and Hu et al. [20] found that the volume effect contains stress.
superposition, the acoustic softening and the dynamic impact. In our previous research, Long et al. [21] proved that the high-frequency vibration is beneficial for the reduction of friction and forming forces during the incremental sheet forming process. Therefore, it is essential to explore the effect of the ultrasonic vibration on the plastic deformation mechanism.

Since the acoustic softening can markedly soften the sheet materials, some researchers have focused on the mechanism of the softening effect. Siddiq et al. [22] built the material model by modifying the variation porous plasticity model to account for instantaneous softening when ultrasonic energy is applied during deformation. Yao et al. [23] proposed a unified acoustic plasticity model to predict the stress–strain relation by analyzing the acoustic softening and acoustic residual hardening effects. Furthermore, the acoustic softening is connected with material flow and some investigation of the influence of ultrasonic vibration on plastic flow has been proposed. Biswas et al. [24] observed that the ultrasonic vibration could improve the material plastic flow in which the material was refined. They proved that the ultrasonic vibration is related to the reduction of the material flow stress. Ahmadi et al. [25] found that the plastic behavior of pure aluminum is remarkably influenced by the ultrasonic vibration, resulting in the reduction of flow stress. Zhao and Liu [26] claimed that the loss of flow stress is closely related to the plastic flow area, which is explained through the proposed material flow model in the ultrasonic roller burning process.

Meanwhile, it was revealed that the ultrasonic vibration drives the forming tool acting on the forming region with the form of dynamic impact, which generates stress waves, activates dislocations and promotes the further plastic flow of the material. Vahdati et al. [27] reported that the dynamic effect of vibration is closely related to material flow and the surface effect during the ultrasonic-assisted incremental forming process. He claimed that the reduction of the static flow stress is decreased by the superimposed oscillatory stress. In addition, Hu et al. [20] found that the additional surface plastic deformation induced by larger ultrasonic energy enhanced the ultrasonic dynamic impact, resulting in the reduction of the forming stress. Meanwhile, researches have shown that the separation effect [28] induced by the dynamic impact of ultrasonic vibration contributes to the reduction of the cutting force [29], the improvement of the surface quality [30] and the increase of the cutting efficiency [31]. After applying the ultrasonic vibration, the separation effect between the forming tool and material sheet could be observed. Patil et al. [32] found that discontinuous contact is one of the main reasons for the decrease in burnishing forces. The separation effect means that the material sheet and the forming tool are not always contacting with each other, which is reported to be beneficial for force reduction, formability enhancement and surface quality improvement [32,33]. Duty cycle (DC) is one of the most important parameters describing the separation effect, which is defined by the ratio of the net contacting duration and the entire duration of one vibration cycle.

However, the material plastic flow during the ultrasonic-assisted incremental sheet forming (UISF) process has rarely been researched by far. Compared with the existing work, our main contributions are as follows.

1. The material flow and deformation behavior of the ultrasonic-assisted straight groove forming process are investigated through three approaches—analytical modeling, finite element (FE) modeling and experimental testing.
2. Both analytical and FE models are established to explore the influence of ultrasonic amplitude and frequency on material flow characteristics. The separation effect for different amplitudes and frequencies is analyzed through finite element modeling.
3. The straight groove tests are performed for the validation of the proposed models. Meanwhile, the effect of the ultrasonic vibration on the microstructure of forming surface is investigated by EBSD technology.

2. Methods

2.1. The principle of UISF process

During the ultrasonic-assisted incremental sheet forming process, the high-frequency vertical vibration is applied to the forming tool head. The schematic diagram of UISF process is presented in Fig. 1.

Similar to the conventional incremental sheet forming, the sheet is fixed on the experimental rig by the blank holder and backing plate. When the forming tool moves along the scheduled tool path, the sheet is formed from the initial metal shape to the final designed shape. High-frequency vibration is employed in the ultrasonic-assisted incremental sheet forming process as an auxiliary energy field on the basis of conventional ISF process. The ultrasonic vibrating system includes a transducer, an amplitude amplifier pole, a forming tool, and associated accessories. The ultrasonic generator produces high-frequency electrical signals to the piezoelectric transducer by which the mechanical vibration can be induced. Then, this vibration is transmitted to the forming tool head through amplification using the amplifier pole. The vibration is as along the tool axial direction (Z-direction) as shown in Fig. 1. Therefore, during the UISF process, the forming tool and the metal sheet exhibit a periodic discontinuous contact state due to the ultrasonic vibration.

The incremental forming processes for straight grooves are selected for detailed investigation. To reveal the effect of ultrasonic vibration on the material flow during the process, analytical models for calculating material plastic flow were established for the straight groove forming. Specifically, different tool paths were investigated which are shown in Fig. 2. For the zigzag-type tool path, the forming tool moves along both horizontal and vertical directions simultaneously during each forming step. By contrast, as for the ladder-type tool path, the forming tool only moves downward at the end of each forming step. Based on our previous experimental work, the experimental parameters are determined as listed in Table 1.

2.2. Theoretical modeling

2.2.1. General assumptions

Based on the periodic nature of the ultrasonic vibration, the variation of material flow in one single vibration period is
analyzed by the theoretical model, which is established on the following assumptions:

- Since the material deformation is dominated by the plastic deformation during each forming step, the elastic deformation is ignored for simplicity.

- Only the material flow within the contact region between the forming tool and the sheet is investigated, while the material flow at the non-contact region is ignored.

- Since the material flow area and deformation caused by ultrasonic vibration is much less than the sheet thickness, the impact of the thickness on the material flow is ignored.

- The forming tool is assumed as a non-damping system with two degrees of freedom.

As shown in Fig. 3, the actual material flow can be discretized by a series of two-dimensional material flow section along the X-direction. It can be observed from Fig. 3(d) that the actual processing depth of different sections is directly influenced by the section angle γ, as given by Eq. (1),

\[ h_p = h_{\text{p}} - R \times R \cos \gamma \]  

Table 1 – The basic parameters of the groove forming processes.

<table>
<thead>
<tr>
<th>Method</th>
<th>Vibration frequency (kHz)</th>
<th>Vibration amplitude (μm)</th>
<th>Groove length (mm)</th>
<th>Step down (mm)</th>
<th>Velocity (mm/min)</th>
<th>Tool diameter (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without vibration</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>2</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td></td>
<td>23</td>
<td>4</td>
<td>50</td>
<td>0.1</td>
<td>800</td>
<td></td>
</tr>
<tr>
<td>With vibration</td>
<td>26</td>
<td>6</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>29</td>
<td>8</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>10</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>
2.2.2. **Modeling for the zigzag-type tool path**

As previously discussed, the actual material flow volume during the straight groove test can be discretized into the material flow area at different two-dimensional sections. In particular, the material flow area at the X-Z section is investigated during the conventional incremental sheet forming process and ultrasonic-assisted incremental sheet forming process. The material flow for the zigzag-type tool path is firstly modeled. To describe the relation of material flow in one vibration cycle (7) precisely, the state of the forming tool and sheet at the i\text{th} and (i+1)th vibration cycle are analyzed. Fig. 4(a) and (b) illustrate the material flow area for the zigzag-type tool path without and with ultrasonic vibration, respectively.

The circular curves in Fig. 4 represent the surface contours of the forming tool at different moments. Specifically, the dashed circular curves and the solid circular curves represent the starting and the ending position of the forming tool for the i\text{th} and (i+1)th vibration cycle, respectively. It should be noted that the starting position of the (i+1)th vibration cycle is the ending position of the i\text{th} vibration cycle. The long dashed curves in Fig. 4(b) indicate the tool position with ultrasonic vibration. The solid lines and the dashed lines represent the unformed and formed surfaces, respectively.

According to Fig. 2, it can be seen that the \( h_p \) is defined by the vertical distance (\( h_a \)) between two forming steps at point A when forming the straight groove with the zigzag-type tool path. It is noted that the \( h_p \) changes along the forming process, which is different during the forming process, as given in Eq. (2).

\[
h_p = \frac{2\Delta z V t}{L_g}
\]

where \( \Delta z \) is the step-down size between two forming steps; \( V \) is the feed rate; \( L_g \) is the length of the straight groove.

During the conventional incremental sheet forming process, when the forming tool moves from the point \( A_i \) to point \( B_i \), the different material flow areas at different section angle \( \gamma \) is defined by zone \( A_{\text{win}}(\gamma) \). To facilitate the calculation, a coordinate system is created, in which the origin is point \( B_i \) and \( X \)-axis is defined along the material surface are displayed in

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**Fig. 3** - The schematic of the contact station with different sections: (a) the three-dimensional of the groove test; (b) the X-Z section; (c) the Y-Z section; (d) the schematic of the processing depth with different section angles.
From Fig. 4(b), it can be seen that the forming tool moves along a sinusoid curve through points $A_i$, $C_i$, $E_i$ to point $B_i$. Compared with the forming process without ultrasonic vibration, the material at zone $A_{ui}(y)$ is a distinct part induced by ultrasonic vibration. It should be noted that an overlapping zone $A_{uwb}(y)$ is created by the zone $A_{ui2}(y)$ at $i$th vibration cycle and the zone $A_{ui1}(y)$ at $(i+1)$th vibration cycle. Therefore, the overlapping zone $A_{uwb}(y)$ should be deducted for the calculation of the material flow area in one vibration cycle.

$$A_{uwb}(y) = A_{ui2}(y) - A_{uwb}(y)$$  \hspace{1cm} (5)$$

where the $A_{uwb}(y)$ is the extra material flow area induced by ultrasonic vibration. Then, the total material flow area during UISF consists of the zone $A_{ui1}(y)$ and the zone $A_{uwb}(y)$, as given by,

$$A_{ui}(y) = A_{ui1}(y) + A_{uwb}(y)$$  \hspace{1cm} (6)$$

where $t$ varies from 0 to $T$, $A_m$ and $f$ are the ultrasonic amplitude and frequency.

$$A(t) = A_m \sin(2\pi ft)$$  \hspace{1cm} (4)$$

where $R$ represents the radius of the forming tool; $h_1$ represents the vertical distance between point $A_i$ and $B_i$ in one vibration cycle, which can be calculated by Eqs. (1) and (2) when the processing time is $T$; $\theta$ is the inclined angle of the zigzag-type tool path; $h_2$ is the processing depth at the section, which can be calculated by Eq. (1). $X_{M1}$, $X_{Mi}$ and $X_{Di}$ represent the abscissa of points $M_i$, $N_i$, $D_i$ in the $i$th vibration cycle, respectively.

The ultrasonic vibration is expressed by Eq. (4),

$$A(t) = A_m \sin(2\pi ft)$$  \hspace{1cm} (4)$$

Fig. 4(c). The area of the zone $A_{ui}(y)$ can be derived with Eq. (3),

$$A_{ui1}(y) = \int_{X_{M1}}^{X_{M2}} \int_{-\sqrt{R^2-x^2}}^{h_1} \frac{\sqrt{x^2 + h_1^2}}{\tan \theta} + R + h_1 \, dz \, dx$$  \hspace{1cm} (3)$$

where $R$ represents the radius of the forming tool; $h_1$ represents the vertical distance between point $A_i$ and $B_i$ in one vibration cycle, which can be calculated by Eqs. (1) and (2) when the processing time is $T$; $\theta$ is the inclined angle of the zigzag-type tool path; $h_2$ is the processing depth at the section, which can be calculated by Eq. (1). $X_{M1}$, $X_{Mi}$ and $X_{Di}$ represent the abscissa of points $M_i$, $N_i$, $D_i$ in the $i$th vibration cycle, respectively.

From Fig. 4(b), it can be seen that the forming tool moves along a sinusoid curve through points $A_i$, $C_i$, $E_i$ to point $B_i$. Compared with the forming process without ultrasonic vibration, the material at zone $A_{ui}(y)$ is a distinct part induced by ultrasonic vibration. It should be noted that an overlapping zone $A_{uwb}(y)$ is created by the zone $A_{ui2}(y)$ at $i$th vibration cycle and the zone $A_{ui1}(y)$ at $(i+1)$th vibration cycle. Therefore, the overlapping zone $A_{uwb}(y)$ should be deducted for the calculation of the material flow area in one vibration cycle.
The material flow area for UISF can be derived by Eqs. (7)--(9),

\[
A_{ui1}(y) = \int_{X_{Ai}}^{X_{Mi}} dx \int_{0}^{h_{y}} \sqrt{R^{2} - \left(x + \frac{h_{1}}{\tan \theta} \right)^{2}} + R - h_{1} \, dz
\] (7)

\[
A_{ui2}(y) = \int_{X_{Ai}}^{X_{Bi}} dx \int_{0}^{h_{y}} \sqrt{R^{2} - \left(x + X_{EB}Y_{EB} \right)^{2} + R - Y_{EB}} \, dz
\] (8)

\[
A_{ui}(y) = \int_{X_{Mi}}^{X_{Bi}} dx \int_{0}^{h_{y}} \sqrt{R^{2} - \left(x + X_{EB}Y_{EB} \right)^{2} + R - Y_{EB}} \, dz
\] (9)

where \(X_{Ai}, X_{Bi}, X_{Pi}, X_{Gi}, X_{Hi}\) and \(X_{Bi}\) are abscissas of points \(M_{i}, D_{i}, F_{i}, P_{i}, G_{i}, H_{i}\) and \(J_{i}\) in ith vibration cycle; \(X_{Bi+1}\) and \(X_{Bi+1}\) are abscissas of the points \(F_{i+1}\) and \(D_{i+1}\) in \((i + 1)\)th vibration cycle; \(X_{EB}\) is the horizontal distance from point \(E_{i}\) to point \(B_{i}\), defined by Eq. (10); \(Y_{EB}\) is the vertical distance between point \(E_{i}\) and \(B_{i}\), defined by Eq. (11),

\[
X_{EB} = \frac{1}{4} \times V \times t_{EB}
\] (10)

\[
Y_{EB} = A_{m} - \frac{1}{2} h_{1}
\] (11)

where \(t_{EB}\) is the time between the forming tool moves from point \(E_{i}\) to point \(B_{i}\).

2.2.3. Modeling for the ladder-type tool path

Similarly, material flow behavior for the ladder-type tool path is also analyzed. Fig. 5(a) and (b) depict the material flow relation in the ladder-type tool path during the conventional ISF and UISF, respectively. The implication of curves and lines is similar to that of Fig. 4. During each forming step with the ladder-type tool path, the \(h_{0}\) is the step-down size (\(\Delta z\)). The value of \(h_{0}\) can be calculated by Eq. (1).

In the conventional ISF process, when the forming tool moves from point \(A_{i}\) to point \(B_{i}\), the material flow area is represented with zone \(A_{ui}(y)\). A similar coordinate system is also created, in which the origin point is \(F_{i}\) and \(X\)-axis is defined along the material surface, as shown in Fig. 5(c). The point \(A_{i}\) and point \(B_{i}\) are symmetrical to the \(Z\)-axis. The area of the zone \(A_{ui}(y)\) can be calculated by,

\[
A_{ui1}(y) = \int_{X_{Ai}}^{X_{Bi}} dx \int_{0}^{h_{y}} \sqrt{R^{2} - \left(x + \frac{1}{2} L_{AB} \right)^{2}} + R \, dz
\] (12)

\[
A_{ui2}(y) = \int_{X_{Bi}}^{X_{Bi}} dx \int_{0}^{h_{y}} \sqrt{R^{2} - \left(x - \frac{1}{2} L_{AB} \right)^{2}} + R \, dz
\] (13)

where \(L_{AB}\) is the horizontal distance between point \(A_{i}\) and \(B_{i}\), which can be obtained by,

\[
L_{AB} = V \times T
\] (13)

where \(T\) is the period of the ultrasonic vibration.

As can be seen from the diagrams in Fig. 5(b) that the forming tool moves through five points \(A_{i}, C_{i}, F_{i}, E_{i}, B_{i}\) along a sinusoidal trajectory under the ultrasonic vibration. Similar to the zigzag-type tool path, the material at zone \(A_{ui2}(y)\) is a distinct part caused by ultrasonic vibration. Also, an overlapping area \(A_{ui}(y)\) is created by the zone \(A_{ui2}(y)\) at the \(i\)th vibration cycle and the zone \(A_{ui}(y)\) at the \((i + 1)\)th vibration cycle and \(A_{ui}(y)\) is deducted for the calculation of material flow area, as given in Eqs. (5) and (6). Then, the material flow area under ultrasonic vibration can be calculated by Eqs. (14)--(16),

\[
A_{ui1}(y) = \int_{X_{Ai}}^{X_{Bi}} dx \int_{0}^{h_{y}} \sqrt{R^{2} - \left(x + \frac{1}{4} L_{AB} \right)^{2}} + R \, dz
\] (14)

\[
A_{ui2}(y) = \int_{X_{Bi}}^{X_{Bi}} dx \int_{0}^{h_{y}} \sqrt{R^{2} - \left(x - \frac{1}{4} L_{AB} \right)^{2}} + R \, dz
\] (15)
\[ A_{uib}(\gamma) = \int_{X_{fi}}^{X_{fj}} dx \int_{0}^{\frac{5}{4}L_{AB}} \sqrt{R^2 - \left(x - \frac{1}{4}L_{AB}\right)^2 + R^2 - Y_{EB}} \, dz \]
\[ + \int_{X_{fj}}^{X} dx \int_{0}^{\frac{5}{4}L_{AB}} \sqrt{R^2 - \left(x - \frac{1}{4}L_{AB}\right)^2 + R^2 - Y_{EB}} \, dz \]
\[ V_{ui2} = 2 \int_{0}^{\alpha} RA_{ui}(\gamma) \, dy \]
\[ V_{uib} = 2 \int_{0}^{\alpha} RA_{uib}(\gamma) \, dy \]

where \( Y_{EB} \) is equal to the ultrasonic amplitude \( A_m \) since there is no vertical increment between points A and B for the ladder-type tool path.

After the material flow area with different section angles was finished, the material flow volume of the material flow region with a zigzag-type tool path or ladder-type tool path can be calculated by Eqs. (17) and (18),

\[ V_{ui} = V_{ui1} + V_{ui2} \]
\[ V_{uie} = V_{ui2} - V_{uib} \]

where \( V_{uie} \) is the extra material flow volume induced by ultrasonic vibration; \( V_{ui} \) is the distinct part induced by ultrasonic vibration; \( V_{ui1} \) is the overlapping volume between the \( i \)th vibration cycle and the \((i+1)\)th vibration cycle; \( V_{ui1} \) is the material flow volume not induced by ultrasonic vibration during UISF. Their values can be obtained by Eqs. (19)-(21),

\[ V_{ui1} = 2 \int_{0}^{\alpha} RA_{ui}(\gamma) \, dy \]

\[ \alpha = \cos \left( \frac{R - h_{r}}{R} \right) \]

2.3. Finite element modeling

In this paper, the material flow behavior and the separation phenomenon at the contact region between the forming tool and the material sheet were explored by FE models using Abaqus/Explicit. The forming process in one vibration period was simulated based on the periodic characteristic of the ultrasonic vibration. Since the contact condition and the plastic deformation at each X-Z section are similar and symmetric about the centerline for the groove forming process, two-dimensional FE models were established.

The forming tool was defined as a rigid body and was modeled with a semicircle for a radius of 2.5 mm. The sheet was modeled as a rectangle with a dimension of 70 × 0.5 mm. The settings of the ultrasonic vibration frequency and the amplitude are listed in Table 1, and the step-down size is 0.1 mm. The surface to surface (explicit) was selected as the contact interaction type between the forming tool and the material sheet. To describe the friction condition properly, the kinematic contact method was used for the mechanical constraint.
formulation. According to our previous experimental work, the friction coefficient at the contact surface between the forming tool and sheet was set as 0.1.

How to improve the finite element simulated predicting accuracy is considerable. Choosing a suitable meshing strategy is a crucial method. As shown in Fig. 6(a), a local mesh refinement around the contact region was performed for the sheet, while the rest of the sheet meshed normally. The double bias seed mesh strategy was used at the upper and lower surfaces of material while the single-bias seed method was used at two vertical side edges. The forming tool initially meshed with the CPS3 type triangular element, and the CPS4R type element was selected for the metal sheet. The FE model is composed of 2289 nodes and 2160 elements.

To simulate the material flow behavior in a single vibration cycle, two simulation steps including loading-down and ultrasonic-moving were defined. The loading-down step was defined to perform the sheet to achieve a stable forming process. The ladder-type tool path was used as the simulated tool path since the vibrating direction is parallel to the normal direction of the material surface. With the good processing and forming characteristic, the AA-1050-O aluminum alloy was selected. And Table 2 presents the material properties. In particular, the constitutive model proposed in our previous research [34] was adopted. This model was based on the crystal plasticity model, in which the ultrasonic softening effect during the UISF process is reflected by adjusting the thermal activation model and the dislocation evolution process.

As for the boundary constraint, the sheet upper surface was seen as the free surface while the other three surfaces were fixed, corresponding to the same boundary condition assumed in the theoretical analysis. It can be observed from Fig. 6 that a path was selected at the top surface for the calculation of the material flow area in the contact area. The material flow area was obtained by calculating the area between two paths between the undeformed state and the deformed state. As for the forming tool, the displacement along the X-axis and Z-axis were defined according to the tool path, and the rotation was fixed. Specifically, to represent the forming path under ultrasonic vibration, the forming tool was defined by superimposing the sinusoidal function to the conventional tool path by the following equations,

\[ f_{\text{ladder}}(x, z) = \begin{cases} 
  z(t) = A_m \sin(2\pi ft) \\
  x(t) = Vt 
\end{cases} \quad (23) \]

\[ f_{\text{zigzag}}(x, z) = \begin{cases} 
  z(t) = -Vt \tan \theta + A_m \sin(2\pi ft) \\
  x(t) = Vt 
\end{cases} \quad (24) \]
2.4. **Experimental setup**

The straight groove test was performed for the evaluation of forming forces and forming properties with ultrasonic vibration. Two types of tool paths described in Fig. 2 were employed for comparison.

**Fig. 7** shows the experimental platform for the UISF process. The vibration amplitude of the forming tool was controlled by the ultrasonic generator, in which the maximum amplitude without loading corresponds to 10 μm. The ultrasonic transducer was connected with the spindle of the machining center, and the sheet to be formed was fixed by a designed fixture. The fixture was mounted on the dynamometer that was installed on the machine table. A dynamometer (Kistler 9257B) with a high-speed signal amplification processor was used to measure the forming force. Since considerable heat can be induced at the transducer under the continuous high-frequency vibration, an air cooling device (CTL-40/0.5) was used to maintain an acceptable working temperature.

The step-down size along Z-direction was set as the value of 0.1 mm and the feed rate along X-direction that parallels to the groove was set as 800 mm/min. The length of the groove was 50 mm. The diameter of the hemispherical forming tool is 5 mm and the rotation speed was set as 0 rad/min during the UISF. The AA-1050-O aluminum sheet was used for the forming tests with the mechanical properties presented in Table 2. The thickness of the sheet is 0.5 mm and was cut into squared shapes with a dimension of 100 mm × 100 mm. A series of experiments listed in Table 3 is performed, in which the effects of the vibration amplitude and the type of tool path are considered.

To investigate the microstructure and texture of the material sheet with and without ultrasonic vibration, the Electron Back Scatter Diffraction (EBSD) technique was employed. The formed area, as small as 3 mm × 3 mm, was extracted from the formed sample by Wire Electric Discharge Machining. The internal surface of the formed part, suffered direct contact with the forming tool, was considered for the analysis.

**Table 3 – The basic parameters for the UISF experiments for straight grooves.**

<table>
<thead>
<tr>
<th>Test no.</th>
<th>Forming type</th>
<th>Vibration amplitude (μm)</th>
<th>Frequency (kHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Ladder</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>Ladder</td>
<td>10</td>
<td>29</td>
</tr>
<tr>
<td>3</td>
<td>Zigzag</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>Zigzag</td>
<td>10</td>
<td>29</td>
</tr>
<tr>
<td>5</td>
<td>Ladder</td>
<td>2.5</td>
<td>29</td>
</tr>
<tr>
<td>6</td>
<td>Ladder</td>
<td>5</td>
<td>29</td>
</tr>
<tr>
<td>7</td>
<td>Ladder</td>
<td>7.5</td>
<td>29</td>
</tr>
</tbody>
</table>

Samples were ground and polished by the MPD-1 machine before testing. Initially, the coarse grinding was performed by emery papers 220#, 500# and 1200# in sequence. Further, the coarse polishing and the fine polishing was executed by 3 μm and 1 μm diamond paste. At last, the OPS (0.04 μm SiO2 suspension liquid) polishing was performed.

3. **Results and discussions**

In the subsequent section, the effects of ultrasonic amplitude, frequency, step-down size and feed rate on material flow characteristics are firstly presented. Then, the separation effect for different tool paths is analyzed. In addition, the experimental results are provided for the verification of the theoretical.

3.1. **Material flow area**

3.1.1. **The effect of amplitude**

The material flow area in the middle section (γ = 0) is the largest, which can be illustrated in Fig. 8(a). Hence, the material flow area in the middle section without and with ultrasonic vibration is further analyzed based on the theoretical model. According to Eqs. (3) and (6), the material flow characteristic can be obtained by calculating the areas of zone $A_{ul}$ and zone $A_{ui}$, which correspond to the total material flow area without
Fig. 8 – The effect of amplitude on the material flow area obtained from theoretical models: (a) the area of \( A_{ui} \) with different sections; (b) the integral display of the material flow area in the middle section \( (\nu = 0) \).

and with ultrasonic vibration, respectively. The total material flow area \( A_{ui} \) during the UISF consists of the zone \( A_{ui1} \) and the zone \( A_{ui2} \). Without ultrasonic vibration, the area of zone \( A_{ui} \) is the same as the zone \( A_{ui1} \), which can be calculated by Eqs. (12) and (14) for the ladder-type tool path. Since the processing depth keeps constant with the ladder-type tool path, the material flow behavior for this type of tool path was analyzed for investigating the effects of ultrasonic amplitude and frequency.

Fig. 8 shows the variation of the material flow area under different ultrasonic amplitudes during the UISF at the frequency of 20 kHz. It can be obtained that the material flow area of \( A_{ui} \) in different sections increases with the increase of the amplitude from Fig. 8(a). Further, it can be observed that both the total material flow area \( (A_{ui}) \) and the vibration-induced flow area \( (A_{ui1}) \) increase with the rise of the ultrasonic amplitude, as shown in Fig. 8(b). Specially, the Fig. 8(b) shows the material flow area in the middle section \( (\nu = 0) \). It is observed that the augment of the material flow area is mainly reflected by the area of zone \( A_{ui2} \), which is much larger than the area of zone \( A_{ui1} \). This is because the value of \( Y_{ui} \) is increased with the increase of the amplitude. By contrast, no obvious change in the area of \( A_{ui1} \). The reason is that the area of zone \( A_{ui1} \) has no direct relationship with the amplitude according to Eqs. (12) and (14). Therefore, these findings suggest that the introduction of UISF can improve the material flow area significantly compared with that of the conventional ISF. However, the material flow area of \( A_{ui} \) at the same amplitude decreases with the increase of section angle, resulting from the reduction of processing depth.

The material flow of zone \( A_{ui2} \) is caused by ultrasonic vibration, which could lead to more materials to participate in the plastic deformation and is considered to be beneficial to the material deformation behavior. Ahmadi et al. [25] and Daud et al. [35] suggested that the high-frequency vibrations can reduce the material flow stress. Further, the ultrasonic vibration decreases the dislocation tangling and pile-ups of the surface grains, resulting in an easier shift of the slip system [36]. As a result, more material flow area is induced under ultrasonic vibration.

Meanwhile, Fig. 9 shows the variation of the material flow volume under different ultrasonic amplitudes during the UISF at the frequency of 20 kHz. It is obvious that the variation trend of the material flow volume is the same as the trend of the material flow area. Therefore, the method that the material flow area was studied in the two-dimensional section for representing the material flow characteristic with ultrasonic vibration in one vibration cycle. It is reasonable.

The material flow behavior in the middle section obtained from the FE model is also analyzed. Fig. 10(a) and (b) present the displacement perpendicular to the surface \( (dz) \) during a single vibration cycle without and with ultrasonic vibration, respectively. As shown in Fig. 10(a), since the time increment for a single vibration cycle is rather small, the variation of \( dz \) at different time points during the process is not obvious. This indicates that the top surface of the sheet always contacts with the forming tool during the process. In comparison, considerable variation can be observed for \( dz \) displacement at different time points with ultrasonic vibration, which suggests that the separation phenomenon happens between the forming tool and the material sheet surface. Specifically,
the dz displacement is smallest at $1/4T$ and largest at $3/4T$ with ultrasonic vibration. The maximum variations of the dz displacement in a single vibration cycle for the forming process without and with ultrasonic vibration are 3.01 $\mu$m and 7.23 $\mu$m, respectively. The interactive impact of the ultrasonic vibration and the spring back of the material sheet contributes to these deviations.

To minimize the influence of the spring back on the results of the material flow area, the simulated result at $3/4T$ (with the largest U2 displacement) was selected for the comparison with theoretical values. The material flow area calculated from the FE model is presented and compared with the theoretically calculated value in Fig. 11. As shown, it is obvious that the increase of ultrasonic amplitude contributes to the increase of the material flow area in both models. There is an average of 4.6% deviation between theoretical and simulated values of the multiple of increase. It is within the acceptable range. Nevertheless, the calculated values of the material flow area with ultrasonic are smaller than those of the simulation.

One of the reasons for the deviation is that the FE model is a kinematical analysis while the theoretical calculation is a simplified static analysis. Specifically, the effect of dynamic impact (ultrasonic peening) of the forming tool is not considered in the theoretical modeling. At the time of $1/2T$ when the forming tool starts to contact with the sheet, large stress can be induced at the sheet surface as shown in Fig. 12. In addition, at the time between $1/2T$ and $3/4T$, extra deformation of the material sheet can result which may also lead to the prediction deviation. The dashed lines in Fig. 11 depict the trend of the times-of-increase of the material flow area under different ultrasonic amplitudes. It can be seen that the times-of-increase in both models rises considerably with the raise of ultrasonic amplitude, which has a direct influence on the deformation behavior of the sheet. Specifically, the material flow area can be increased by 12 times with the amplitude of 10 $\mu$m compared with that without vibration according to the FE simulation results.

Fig. 10 – The variation of the displacement parallel to the tool axis (dz) in one vibration cycle at different step times from FE simulated models: (a) without vibration; (b) with vibration.

Fig. 11 – The comparison of the effect of amplitude on the material flow area and the multiple of increase.
3.1.2. The effect of frequency

The variations of the material flow area under different ultrasonic frequencies during the UISF at the amplitude of 6 μm, as shown in Fig. 13. Similar to Fig. 8(a), Fig. 13(a) presents that the material flow area of \( A_{ui} \) decreases with the increase of both section angle and frequency. Further, the individual display of three material flow areas is revealed in Fig. 13(b). It can be found that the increase of the frequency contributes to the reduction of the total material flow area \( A_{ui} \). The total flow area is contributed by both the non-vibration related zone \( A_{ui1} \) and the vibration-induced area \( A_{ui2} \). As shown in Fig. 13(b), the area of zone \( A_{ui2} \) is projected to increase with the increase of the ultrasonic frequency, while the area of zone \( A_{ui1} \) is decreased. The reason for the decrease of zone \( A_{ui1} \) is that the horizontal moving length of the forming tool \( L_{ui} \) is decreased with the raise of ultrasonic frequency since the period of a single vibration cycle is shortened. As for the vibration-induced flow area, the increase of zone \( A_{ui2} \) is attributed to the rise of the horizontal deviation between the forming tool surfaces at 3/4T and T with the increase of ultrasonic frequency according to Eqs. (5), (15) and (16). However, this horizontal deviation is smaller than the reduction range of \( L_{ui} \). Since the area of \( A_{ui} \) is the interactive effect of zone \( A_{ui1} \) and zone \( A_{ui2} \), the total material flow area \( A_{ui} \) will decrease with the rise of the ultrasonic frequency. In addition, as shown in Fig. 13, the values of the non-vibration related material flow area \( A_{ui1} \) and the vibration-induced material flow area \( A_{ui2} \) are 615.65 μm² and 4088.42 μm² at the frequency of 20 kHz, respectively. The material flow area with vibration is 6.27 times than that without vibration. It is concluded that the material flow area is dramatically increased with ultrasonic vibration. However, the total material flow area \( A_{ui} \) only shows a slight decrease rate of 3.1% with the frequency ranging from 20 kHz to 29 kHz.

It can be seen from Fig. 14 that, for both models, the material flow area is slightly decreased with the raise of ultrasonic frequency. A similar trend between the theoretical model and the FE model further confirms the reliability of the theoretical model. According to Fig. 5 and Eq. (13), the vibration frequency directly affects the horizontal displacement of the forming tool during one cycle. Although the total material flow area decreases with the rise of the ultrasonic vibration frequency, the variation is not considerable. Consequently, according to Eq. (15), the effect of frequency on the material flow area is

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Fig. 12 – The stress along the thickness direction (S22) at the initial contacting point from the FE models.

Fig. 13 – The effect of frequency on the material flow area obtained from theoretical models: (a) the area of \( A_{ui} \) with different sections; (b) individual display of three material flow areas in the middle section \( (\gamma = 0) \).
limited. The above findings agree with experimental observation as has been reported by other researchers [23,37,38].

3.1.3. The influence of step-down size and feed rate
The influence of step-down size and feed rate on the material flow area are also explored. To make the comparison fair, the position with the same processing depth at the forming pass (the middle point) is examined. Fig. 15 shows the influence of step-down size and feed rate on the material flow with two different tool paths, respectively.

As shown in Fig. 15(a), the material flow area increases as the step-down size increases for both zone $A_{\text{ul}}$ (total material flow area in UISF) and zone $A_{\text{uis}}$ (material flow area in ISF). However, the increased rate of material flow area in UISF and ISF is different. Specifically, as the step-down size increases from 0.1 mm to 0.3 mm, the material flow area $A_{\text{ul}}$ increases by 203.1% while that is 81.7% for zone $A_{\text{uis}}$. Meanwhile, it can be observed that the increased value of the material flow area $A_{\text{ul}}$ is larger than that of zone $A_{\text{uis}}$. Eq. (14) demonstrates that the decrease of the step-down size results in the rise of the vertical variation of the material flow area without and with ultrasonic vibration. As for the material flow area $A_{\text{uis}}$, the material flow area scales at the same level as the step-down size increases. By contrast, during ultrasonic-assisted forming, although both the non-vibration related material flow area $A_{\text{uis}}$ and the vibration-induced material flow area $A_{\text{uis}}$ increase as the step-down size increases, the increase rate is smaller than that of the conventional ISF. The reason is that the value of the material flow area $A_{\text{ul}}$ is significantly larger than that for $A_{\text{uis}}$ at the same step-down size. Despite that the variation of the material flow area induced by step down occupies a considerable proportion, larger step-down size has a negative effect on the surface finish [39]. By contrast, the ultrasonic vibration is evidenced to have a positive effect on the improvement of the surface quality [40,41].

By contrast, as for the feed rate shown in Fig. 15(b), although the total material flow areas for both cases with and without vibration are increased with increasing the feed rate, the vibration-induced area $A_{\text{uis}}$ is reduced. The increase of the
feed rate will enlarge the moving distance of the forming tool from point A to point B \( (L_{AB}) \), which is a major influencing factor for zone \( A_u \) in ISF. However, this material flow area only takes a small portion of the material flow area in UISF. The decrease of zone \( A_{ue} \) is attributed to the decrease of the deviation between forming tool surfaces at the time of \( 3/4T \) and \( T \) with the increase of feed rate according to Eqs. (5), (15) and (16).

3.1.4. The effect of tool path
As already noticed in Fig. 15, no obvious difference for the material flow area can be observed between the zigzag-type and ladder-type tool paths under the same forming parameters. According to Figs. 4 and 5, the material flow area is related to the vertical distance \( h_1 \) between the starting point A and the ending point B. In particular, vertical distance \( h_1 \) is the distinct difference between the zigzag-type and the ladder-type tool paths while the value of \( L_{AB} \) is the same. Furthermore, according to Eqs. (2), (10) and (11), the value of \( h_1 \) \((2.65e–6)\) is much smaller than the value of \( L_{AB} \) \((6.67e–4)\) at the vibrating frequency of 20 kHz. Therefore, the value of \( h_1 \) has a limited effect on the calculated material flow area.

As discussed above, the difference of the material flow area between two studied paths is relatively small at the middle point for a single vibration period. Hence, the total material flow areas for an entire forming step are further compared with two different tool paths. The calculated results for theoretical models are shown in Fig. 16. For the total material flow area, it can be found from Fig. 16(a) and (b) that the area of zone \( A_{ue} \) caused by the zigzag-type tool path is slightly larger than that of ladder-type without ultrasonic vibration. When the forming tool moves along the zigzag-type tool path, larger material is involved for the plastic deformation due to the continuous increment at the Z-direction compared with the ladder-type tool path. In addition, the material deformation in the zigzag-tool path is gentle during processing. Based on the above analysis, the peak force is smaller at the turning point for the zigzag-tool path which leads to a relatively uniform axial force variation [42]. This can also be observed from our experimental results in Fig. 22. However, in UISF, the material flow area of zone \( A_{ui} \) caused by the zigzag-type tool path is smaller than that of the ladder-type tool path, as given in Fig. 16(c) and (d). After calculating the difference of the material flow area between ladder-type and zigzag-type, the rate of

![Fig. 16 – The comparison of the total material flow area with different types of tool path at the frequency of 20 kHz and the amplitude of 6 \( \mu \)m from the theoretical models: (a) the effect of step-down size on material flow area without vibration; (b) the effect of feed rate on material flow area without vibration; (c) the effect of step-down size on material flow area with vibration; (d) the effect of feed rate on material flow area with vibration.](image-url)
change can be defined as the ratio between the difference and the flow area with zigzag-type. It can be obtained from Fig. 16 that the average value of the rate of change with a ladder-type tool path is larger than that with a zigzag-type tool path under both the same step-down size and the feed rate.

It is observed that the ultrasonic vibration has a more prominent effect on the material flow area with a ladder-type tool path than that with a zigzag-type tool path. The reason is that the ultrasonic vibrating direction is parallel to the normal direction of the material surface in the ladder-type tool path since the processing depth keeps constant at a certain forming pass. The ultrasonic vibration direction is parallel to the normal direction of the contact surface between the forming tool and the material sheet. In this case, the effect of the ultrasonic vibration can be maximized. By contrast, a deflection angle is produced between the ultrasonic vibrating direction and the normal direction of the material surface with the zigzag-type tool path, which weakens the utilize of the ultrasonic vibration.

In order to explain the relationship between the material flow deformation and the flow stress with ultrasonic vibration, the state of Von Mises stress obtained from FE models at the initial contact point is discussed. As shown in Fig. 17, the red line represents the smoothed results of the original data in USIF. It is noted that the average value of the Mises stress in USIF is smaller than that in ISF, which indicates that the Mises stress is decreased by adding ultrasonic vibration. The reason is that the material flow area contributes to the reduction of the flow stress at a unit time [26,36]. The impact of ultrasonic vibration on the change of material flow area is further discussed later based on our experimental results presented in Figs. 22, 23 and 25.

3.2. The separation effect

Based on the assumptions in Section 2.2.1, the contact conditions in one vibration cycle with zigzag-type and ladder-type tool paths are obtained in Fig. 18(a) and (b). With the applied high-frequency vibration, the forming tool moves from point A through C and E to point B in one vibration cycle. During the process, the forming tool contacts with the sheet within A′B′, while the separation is occurred within ACA′. The line PP′ means the sheet surface level at the starting of the vibration cycle and the line OO′ represents the sheet surface level at the middle position of the vibration curve in zigzag-type tool path as illustrate in Fig. 18(a). However, the lines of PP′ and OO′ are overlapped for the ladder-type tool path in Fig. 18(b). Therefore, the duty cycle can be given by Eq. (25) for both tool paths.

\[
DC = \frac{A'B'}{AB}
\]  

(25)

The values of a duty cycle for two different tool paths are calculated based on the theoretical model, which are 0.4625 and 0.499 for the ladder-type and zigzag-type tool paths, respectively. Since the theoretical model is based on static analysis and the elastic deformation is ignored for simplicity, the calculated duty cycle values for different amplitudes keep the same. To further investigate the effects of amplitude and frequency on the duty cycle, the results obtained from FE simulations are analyzed.

In order to obtain the duty cycle in the FE models, the evolution of the different position of the forming tool head and the sheet surface is analyzed. Fig. 19(a) and (b) present the effects of amplitude and frequency on the value of the duty cycle obtained from FE models. It can be observed that the duty cycle decreases as increasing both the amplitude as well as the frequency, which indicates that the separation period during one cycle is increased. This separation effect was proved to have a positive effect on the reduction of the friction force [37]. Meanwhile, Biswas et al. [24] and Patil et al. [32] reported that the separation effect can decrease the forming force. The findings provide a foundation for the investigation of the forming force and the contact condition in USIF, which is further discussed later in our experimental results.

3.3. Experimental verification and discussions

The above analyses suggest that the introduction of ultrasonic vibration helps to increase the material flow area and reduces the contact period, resulting in the reduction of forming force. This section provides experimental evidence for the aforementioned findings.

![Fig. 17 – The change of the flow stress at the initial contact point with and without ultrasonic vibration from FE models.](image)

![Fig. 18 – Contact condition for two tool paths: (a) the zigzag-type tool path; (b) the ladder-type tool path.](image)
Fig. 19 – The DC at the initial contact point obtained from FE models: (a) the impact of amplitude on duty cycle; (b) the impact of frequency on the duty cycle.

Fig. 20 – The comparison of the forming forces during groove forming tests with and without ultrasonic vibration.

Fig. 21 – The effect of the amplitude on the force reduction rate of vertical forming force.

Fig. 20 compares the forming forces during groove forming tests with and without ultrasonic vibration (tests no. 3 and 4) under the zigzag-type tool path. At each forming pass, both vertical and horizontal forces gradually rise as the forming tool approaches to the end of the groove. Then, both force components experience a sharp increase at the turning points. A detailed explanation of forming forces was provided in our previous work [43]. It should be pointed out that the forces for both directions are reduced significantly once the ultrasonic vibration is applied.

Fig. 21 presents the effect of amplitude on the force reduction rate at the end stage with the ladder tool path (tests no. 1, 5, 6 and 7). It is observed that the reduction rate of forming force is larger for higher vibration amplitude, which agrees with the same trend as the effect of amplitude on DC as shown in Fig. 19(a). This trend is also similar to that of the material flow area derived from both analytical and finite element models. As Shi et al. [44] reported, the superimposed ultrasonic vibration energy is increased with the increase of the amplitude. The material plastic flow is mainly related to the motion of dislocations [45], and the superimposed ultrasonic vibration energy is significant to lower the energy barrier of the dislocation motion, thus the material flow area is increased. As a result, the material is softened, which means that the material will be softened. This variation plays a key factor in the reduction of the forming force (Fig. 20) and the improvement of the formability (Fig. 26). In conclusion, this same trend indicates that separation effect and the material flow area during UISF could lead to the reduction of forming force. In particular, the reduction of vertical forming force can reach to 40.09% at the vibration amplitude of 10 \( \mu \text{m} \).

Fig. 22(a) depicts the comparison of the vertical force with different tool paths and Fig. 22(b) shows the enlarged view of vertical forces of a typical forming pass. It can be observed that the forming time with the zigzag-type tool path is relatively smaller than that of the ladder-type tool path to reach the same forming depth. This phenomenon is determined by the characteristic of the continuous increment along the thickness direction with the zigzag-type tool path, which leads the machine tool to move in two feed axes simultaneously. As shown in Fig. 22(b), the points A, B, C represent the start, the
The comparison of the vertical forces with two tool paths: (a) the overall trend; (b) the enlarged view for a single forming pass.

Comparison of the vertical force at different positions with different tool paths: (a) the start stage; (b) the middle stage; (c) the end-stage.

middle and the end positions in a forming pass. It can be seen that the peak force with the zigzag-type is smaller than that of ladder-type. Meanwhile, when the forming tool moves from point A to point B, the forming force with zigzag-type is smaller than that in ladder-type. This is because the processing depth at these regions is smaller than that of ladder-type, which can be obtained in Eq. (2). As a result, the forming force of the zigzag-type tool path is larger in the second half of the forming pass due to the increased processing depth. Nevertheless, it can also be observed that the forming force is decreased after applying the ultrasonic vibration.

To further investigate the evolution of forming forces of different tool paths with and without ultrasonic vibration, vertical forces are selected at every 10 forming passes. For each forming pass, three points located at the start stage (A), the middle stage (B) and the end-stage (C) are selected. Compared with that of points B and C, the difference of vertical force between two tool paths is more prominent. In both points, the reduction of vertical force with ultrasonic vibration is noteworthy. In the middle stage, it can be observed in Fig. 23(b) that the vertical forming force of the zigzag-type tool path is smaller than that of the ladder-type tool path without ultrasonic vibration. However, the vertical forming force of the ladder-type tool path is smaller than that of the ladder-type tool path with ultrasonic vibration. The reason can be explained by the variation of the material flow area, as depicted in Fig. 18(c) and (d). At the same processing depth, the vertical forming force is smaller due to the increased material flow area. In addition, as depicted in Fig. 24, the force reduction rate of the ladder-type tool path is larger than that of the zigzag-type tool path at the
Fig. 25 – Comparison of the horizontal force with different tool paths: (a) the horizontal forming force in UISF and ISF; (b) the forming force with different forming passes at the middle stage; (c) the force reduction rate.

Fig. 26 – The outer surface of the formed groove at the ends: (a) the zigzag-type tool path without ultrasonic vibration; (b) the ladder-type tool path without ultrasonic vibration; (c) the ladder-type tool path without ultrasonic vibration; (d) the ladder-type tool path with ultrasonic vibration.

middle stage. This agrees with the trend in Fig. 18 that the increased rate of the material flow area with a ladder-type tool path is larger than that with a zigzag-type tool path.

In addition, the horizontal forming force is compared. Fig. 25(b) depicts the comparison of the horizontal forming force at different moments with different tool paths. It can be observed that the horizontal forming force of the ladder-type tool path is larger than that of a zigzag-type tool path without ultrasonic vibration. With the adoption of the ultrasonic vibration, the forming force is reduced with the ladder-type tool path. The variation trend of horizontal forces is the same as that of vertical forces. Specifically, as shown in Fig. 25(c), it can be observed that the reduction rate of the horizontal force of the zigzag-type tool path is smaller than that of the ladder-type tool path in the middle position.

The formed parts with two different tool paths with and without ultrasonic vibration are examined. It can be found in Fig. 26(a) and (b) that the outer surfaces at the ends of the groove formed with zigzag-type tool path are intact for both cases with and without vibration. However, the necking is observed at the outer surface for the part formed with the ladder-type tool path without ultrasonic vibration while the surface is intact for the part formed with vibration, as depicted in Fig. 26(c) and (d). Since the material flow area with the zigzag-type tool path is larger than that of the ladder-type without ultrasonic vibration from Fig. 16(a) and (b), more materials participate in the plastic deformation, which leads to the evener deformation and the improved formability. Consequently, it can be concluded that the formability is improved after the application of ultrasonic vibration. In addition, the formability by using the zigzag-type is better than that using ladder-type without ultrasonic vibration.

Moreover, the colored normal direction IPFs map and the grain boundary map with the calculated fraction of misorientation angles for undeformed sheet and the deformed sheet without and with ultrasonic vibration in ISF are illustrated in Fig. 27. It can be observed that the grain size becomes more uniform after the incremental sheet forming process. Meanwhile, a number of equiaxed grains with an average size of 1 \( \mu m \) are generated. This may due to the repeated compaction between the forming tool and the material sheet, resulting in broken of the large grains. However, it can be found that the assistance of the ultrasonic vibration increases the grain size to about 1.2 \( \mu m \). This can be explained by the static/dynamic recovery below the dynamic recrystallization temperature due to the superimposed ultrasonic vibration energy. This could also reduce strain hardening rate and soften the material, resulting in the increase of the material flow area.

Subsequently, misorientation histograms and grain boundary maps are also examined. It can be seen that high angle grain boundaries is increased while low angle grain boundaries is decreased after the incremental sheet forming process. However, after the ultrasonic vibration was applied, the fraction of the low angle grain boundaries is greatly enhanced. The reason is that the superimposed ultrasonic vibration lower the energy barrier for the dislocation motion, resulting in more uniform dislocation motions. This also indicates that the ultrasonic vibration could improve the material flow and deformation behavior during the ISF process.
4. Conclusions

In this paper, the material flow and deformation behavior of the UISF process are investigated through three approaches: analytical modeling, finite element modeling, and experimental testing. The main conclusions are drawn as follows:

- The material flow area is significantly increased after applying ultrasonic vibration, leading to more materials participating in the plastic deformation. Ultrasonic vibration can effectively reduce the stress along the thickness direction and the Von Mises stress. This reduction plays a significant effect in the reduction of the forming force and the improvement of the formability.

- The material flow area increases considerably with the increase of the ultrasonic amplitude, while it decreases slightly with the raise of ultrasonic frequency. The material flow area increases with the increase of the step-down size and feed rate for both cases with and without ultrasonic vibration.

- The reduction of forming forces in UISF is attributed to the reduced flow stress due to the increased material flow area and the reduced contact time due to the separation effect caused by the dynamic effect of ultrasonic vibration.

- The small included angle between the ultrasonic vibrating direction and the normal direction of the contact surface has a significant effect on the utilization of the ultrasonic vibration. Particularly, the adoption of the ultrasonic vibration increases the grain size and decreases the misorientation angle of the formed parts.

Although the analytical model provides fundamentals for the investigation of the material flow area and deformation mechanism of the UISF, there are also some limitations: (1) the effect on the material flow of the elastic deformation and the thickness of the material sheet was not considered; (2) the FE simulation is based on a 2D model, which is different from the 3D contact condition in actual situation. Therefore, further efforts could be focused on the development of a three-dimensional FE simulate model considering the elastic deformation and forced vibration of the metal sheets.
Conflicts of interest

The authors declare no conflicts of interest.

Acknowledgments

This work is funded by National Natural Science Foundation of China (51975328, 51605258), Postdoctoral Innovation Project of Shandong Province (201701011), Qilu University of Technology (Shandong Academy of Sciences) International Cooperation Study Special Fund Project (QLUTGJHZ2018006) and Young Scholars Program of Shandong University (2018WJLH55).

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