Original Article

Constitutive equation and microstructure evaluation of an extruded aluminum alloy

Saravanan L*, Senthilvelan T

Department of Mechanical Engineering, Pondicherry Engineering College, Pondicherry 605014, India

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ABSTRACT

The flow-stress behavior of an extruded aluminum alloy has been studied by conducting a set of warm and hot compression tests. The compression tests were carried out in the temperature range of 373 K–773 K and strain rates of 0.001, 0.01 and 0.1 s⁻¹, up to a strain of 0.5. Based on the results obtained from these tests, a mathematical model was obtained to predict flow stress for a given strain. The effect of temperature and strain rate on deformation behavior was ascertained by determining the Zener–Hollomon parameter. The influence of strain has been incorporated by employing an Arrhenius-type constitutive equation, considering the related material constants as functions of strain. The comparison of results indicated good agreement between the predicted and measured flow-stress values in the relevant temperature range. The correlation coefficient and average absolute relative error of the model were found to be 0.9965 and 4.26% respectively confirming good accuracy.

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1. Introduction

Aerospace and automotive industries widely use aluminum alloys for manufacturing the structural components [1]. Comprehensive review of the research work carried out in the high temperature flow behavior of aluminum and its alloys can be found in [2]. Further investigation is required to understand the characteristics of certain aluminum alloys under different processing conditions and forming processes. Extrusion is inevitably used in automotive and structural applications as a preliminary process. The present work has thus been focused on arriving at a relationship between the flow stress, strain, strain rate and temperature to predict the flow behavior of extruded zinc based aluminum alloy. Toward this end, hot compression tests were conducted with a range of strain rates and temperatures. The experimental stress–strain data thus obtained had been employed to derive the constitutive equation relating flow stress, strain rate and temperature incorporating the proper compensation of strain. Finally, the validity of the developed constitutive equation has been examined for the processing conditions considered.
2. Experimental details

2.1. Material

The chemical composition of the commercial aluminum alloy used in this work is given in Table 1. The aluminum alloy has been produced by using a vortex method. The crucible was charged with 1200 g of aluminum alloy, and heated up to 650 °C for melting. The graphite stirrer rod was inserted into the melt, positioned just below the surface of the melt and rotated at 500 rpm. Aluminum alloy slurry was bottom poured into preheated cast iron molds. The alloy was shaped in the form of cylinder with 35 mm outer diameter and a height of 210 mm. Billets of size φ32 mm × 50 mm were machined from these cylindrical rods and these billets were subjected to two stages of extrusion.

2.2. Extrusion of aluminum alloy

Extrusion process was carried out in ENKAY universal testing machine (UTM) of 600 kN capacity. The extrusion die was heated to the required temperature in a pit-type furnace. Once the die had attained the desired temperature, a period of 30 min, was allowed to elapse before the extrusion was carried out. This time is long enough to allow the billet to reach a steady-state temperature. The sizes of the billet material after first and second stage extrusions were φ28 mm × 60 mm and φ24 mm × 85 mm, respectively. The photograph of specimens subjected to different levels of extrusion is shown in Fig. 1 along with the as-cast specimen.

2.3. Hot compression test

In order to determine the stress–strain behavior of the alloy, uniaxial one-hit hot compression test was carried out using precise digital controller-servo equipped universal testing machine. One-hit hot compression test was conducted at different temperatures of 373, 473, 573, 673, and 773 K and different strain rates of 0.001, 0.01 and 0.1 s⁻¹, up to a strain of 0.5. Five samples were tested for each temperature-strain rate combination and mean values of stress and strain were considered for further analysis.

Cylindrical specimens of size φ24 mm × 24 mm were used. Powdered molybdenum disulphide was used as lubricant up to 573 K and powdered graphite was used as lubricant for temperature ranging from 673 to 773 K. These colloidal powders were laid between punch and specimen for minimizing the friction. Samples were then heated to the test temperature. After heating, the samples were held at the test temperature for 5 min and then hot compressed up to a strain of 50% [3]. True stress values were recorded using a pressure transducer with a resolution of 0.5 N. The microprocessor controlled UTM machine is equipped with inbuilt strain gauge for instantaneous strain measurement and interfaced with a Wincom software installed computer. The software generates the load and displacement curve with data points at the specified data logging rate. This information was used to develop the flow curves. The photographic image of the samples, hot compressed at different strain values, is shown in Fig. 2.

| Table 1 – Chemical composition (wt.%) of aluminum alloy. |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| Element        | Zn       | Mg  | Cu  | Fe  | Si  | Cr  | Ni  | Ti  | Al  |
| Measured (wt.%)| 6.09     | 2.68| 1.28| 0.18| 0.13| 0.12| 0.01| 0.01| Bal |

3. Results and discussion

3.1. Flow behavior

The flow stress vs flow strain curves of the aluminum alloy obtained at various temperatures and strain rates up to a height reduction of 50% are shown in Fig. 3. In all these curves, the stress increases linearly with increase in strain up to a particular strain value and thereafter slows down and saturates with or without a slight dip in value. In Fig. 3(a), at 373, 473 and 573 K, the flow stress increases with strain initially and gently approaches a steady value with further increase in strain. The initial increase in stress may be attributed to the increase in dislocation density due to strain hardening. This type of flow curve is referred as dynamic recovery type of flow curve. Dynamic recovery is the only softening mechanism occurring under this process condition [4,5]. In case of 673 and 773 K the curve exhibits a slight dip in flow-stress value after reaching the peak stress. It can also be observed that the flow stress increases initially at a slower rate compared to that at lower temperatures. This happens because of the reason that the dynamic softening dominates over work hardening at higher temperatures [6,7]. The flow curve drops continuously until the balance between work hardening and dynamic softening is achieved. This pattern of the curve indicates the occurrence of dynamic recrystallization. Further, a reduction in stress value can be observed for an increase in temperature. This happens since movement of dislocations is favored by thermal activation at higher deformation temperatures [8,9].

Comparing the curves with same temperature in Fig. 3(a)–(c), it can be observed that the flow stress decreases with decreasing strain rate. As strain rate is decreased, the

![Fig. 1 – Photograph of specimens with different levels of extrusion (left to right: ε = 0, ε = 0.266, ε = 0.305).](image-url)
rate at which dislocations get multiplied is reduced leading to less tangled dislocation structures [10]. Therefore, reduced strain rate is expected to lower the stress that is needed for dislocation movement, and leads to the reduced flow stress.

3.2. Microstructure evolution

Further, the occurrence of dynamic recovery and recrystallization is also confirmed by the optical micrographs. The optical micrograph of as-cast aluminum alloy and extruded aluminum alloy samples, hot compressed at a temperature of 573 K, are shown in Fig. 4(a) and (b) respectively.

Fig. 4(c) and (d) shows the optical micrograph of as-cast aluminum alloy and extruded aluminum alloy samples respectively, hot compressed at a temperature of 773 K. Partially recrystallized grain structure can be observed in Fig. 4(a). The figure reveals regions of fine sub-grains in the grain boundaries as well as in the interior region indicating initialization of dynamic recrystallization [11]. Whereas, in Fig. 4(c), nearly fully recrystallized grain structure is observed. Moreover, the second phase can be observed in both the figures as dark spots within the recrystallized regions indicating the possibility of particle induced nucleation. Fig. 4(b) and (d) reveal flattened and equiaxed grains with relatively high angle grain boundaries compared to as-cast alloy.
3.3. Constitutive equation of flow stress during hot deformation

Constitutive equations usually describe the relation between flow stress and flow strain under variable strain rate and temperature. The flow stress–strain data obtained from one-hit hot compression tests under different strain rate and temperature conditions can be used to determine the material. Furthermore, the effects of temperature and strain rate on material deformation behavior could be expressed by means of Zener–Holloman parameter, $Z$, which is given by Eq. (1).

$$Z = \dot{\varepsilon} \exp \left( \frac{Q}{RT} \right)$$

where, $\dot{\varepsilon}$ is the strain rate (s$^{-1}$), $R$ is the universal gas constant (8.3145 J mol$^{-1}$ K$^{-1}$), $T$ is the absolute temperature (K), and $Q$ is the activation energy (kJ mol$^{-1}$).

The relationship between the flow stress $\sigma$, temperature $T$ and strain rate $\dot{\varepsilon}$ is expressed by the Arrhenius type equation [12],

$$\dot{\varepsilon} = Af(\sigma) \exp \left( - \frac{Q}{RT} \right)$$

The hyperbolic-sine Arrhenius-type equation given by Eq. (3) is applied in a wide range of stress regimes [13].

$$\dot{\varepsilon} = A \left[ \sinh (\omega \sigma) \exp \left( - \frac{Q}{RT} \right) \right]$$

where, $n$ is the stress exponent, $\beta$ and $\alpha$ are stress adjustment factors (MPa$^{-1}$), and $A$ is material constant.

In order to determine the values of $Q$, $A$ and $\alpha$, two parameters $n_1$ and $\beta$ are defined as given by Eqs. (4) and (5).

$$n_1 = \left( \frac{\partial \ln \dot{\varepsilon}}{\partial \ln \sigma} \right)_T$$

$$\beta = \left( \frac{\partial \ln \dot{\varepsilon}}{\partial \sigma} \right)_T$$

The values of $n_1$ and $\beta$ can be obtained from the slope of the $\ln(\sigma)$ vs $\ln(\dot{\varepsilon})$ plot and $\ln(\dot{\varepsilon})$ vs $\sigma$ plot respectively by linear fit method. After this $\alpha$ can be computed using the following relation given by Eq. (6) [14].

$$\alpha = \frac{\beta}{n_1}$$
Graphs used for determining \( n_1 \) and \( \beta \) are shown in Fig. 5(a) and (b). Taking the logarithm on both sides of Eq. (3) and then differentiating gives the expression for the activation energy, \( Q \), as given in Eq. (7).

\[
Q = \ln \left[ \frac{\beta \ln \left(\sin h(\omega)\right)}{\alpha n} \right]
\]

(7)

where, \( n \) is the stress exponent given by Eq. (8).

\[
n = \left[ \frac{\partial \ln \dot{\varepsilon}}{\partial \ln \left(\sin h(\omega)\right)} \right]_T
\]

(8)

The value of \( n \) can be derived from the slopes of the straight line representations of \( \ln[\sin h(\omega)] \) vs \( \ln(i) \) plots at constant temperature, as shown in Fig. 5(c).

The average of the slope values is taken as the \( n \) value. The \( n \) value was found to be 4.6 for the material considered. The values of \( Q \) are obtained in a similar manner from the slope of every single line in the \( \ln[\sin h(\omega)] \) vs \( 1/T \) plots by linear fit method as shown Fig. 5(d). The average value of \( Q \) obtained from the graph is 196 kJ mol\(^{-1}\).

### 3.4. Compensation of strain

The material constants \( \alpha, \beta, n, Q \) and \( A \) are significantly influenced by strain. But, it was usually assumed that the influence of strain on flow stress at elevated temperatures is insignificant. However, it was observed that the deformation activation energy \( Q \) was strongly influenced by the strain in A356 aluminum alloy [15], commercial purity aluminum alloy [4] and 7050 aluminum alloy [6]. Therefore, the compensation of the strain may have a significant effect on the accuracy of the flow-stress prediction and should be taken into account in order to arrive at the proper constitutive equation. Fig. 6 shows the variation of material constants with respect to variation in strain for the material under study. It can be observed that the strain value has significant impact on material constants. Thus to predict the flow stress accurately, the influence of the strain was incorporated into the constitutive equation by assuming that the activation energy \( Q \) and other material constants were polynomial functions of the strain [16,17].

\[
\alpha = a_0 + a_1\varepsilon + a_2\varepsilon^2 + a_3\varepsilon^3 + a_4\varepsilon^4 + a_5\varepsilon^5
\]

\[
n = n_0 + n_1\varepsilon + n_2\varepsilon^2 + n_3\varepsilon^3 + n_4\varepsilon^4 + n_5\varepsilon^5
\]

\[
Q = Q_0 + Q_1\varepsilon + Q_2\varepsilon^2 + Q_3\varepsilon^3 + Q_4\varepsilon^4 + Q_5\varepsilon^5
\]

\[
\ln A = A_0 + A_1\varepsilon + A_2\varepsilon^2 + A_3\varepsilon^3 + A_4\varepsilon^4 + A_5\varepsilon^5
\]

(9)

In this study, the values of the material constants were evaluated at various strains in the range of 0–0.5 at the interval of 0.05. Based on the good fitting correlation and accuracy, a fifth order polynomial, as shown in Eq. (9), was found to represent the influence of strain on those material constants. The coefficients of the polynomial functions \( \alpha, n, Q, \) and \( \ln A \) were provided in Table 2. Considering Eqs. (1) and (3), the constitutive equation to predict the flow stress can be expressed as given in Eq. (10). With the evaluated material constants at a
In particular, the flow stress can be predicted using this equation.

\[
\sigma = \frac{1}{\alpha} \ln \left( \frac{Z}{A} \right)^{1/n} + \left[ \frac{Z}{A} \right]^{(2/n)} + 1 \right]^{1/2}
\] (10)

3.5. Comparison of experimental and predicted results

In order to validate the constitutive equation obtained, a comparison between the experimental and predicted data under different processing conditions is plotted and shown in Fig. 7. From this graph, it can be observed that a good agreement had been obtained between the experimental and predicted stress values. The predictability of the constitutive equation is also quantified employing standard statistical parameters such as correlation coefficient and average absolute relative error (AARE). These are commonly used statistical parameters which provide information about the strength of linear relationship between the observed and the calculated values [18]. The experimental flow stress is plotted against flow stress predicted based on the developed constitutive equation over the entire range of strain, strain rate and temperature as shown in Fig. 8, to analyze the correlation between them. The correlation coefficient and average absolute relative error are found to be 0.9965 and 4.26% respectively. This result confirms the accuracy of the developed constitutive equation with strain compensation for the alloy under study.

### Table 2 - Polynomial fitting results of \( \alpha \), \( n \), \( Q \), and \( \ln A \) of aluminum alloy.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Value</th>
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<th>Coefficient</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_0 )</td>
<td>0.07057</td>
<td>( n_0 )</td>
<td>6.64</td>
<td>( Q_0 )</td>
<td>117.69</td>
<td>( A_0 )</td>
<td>33.69</td>
</tr>
<tr>
<td>( \alpha_1 )</td>
<td>-0.00693</td>
<td>( n_1 )</td>
<td>-12.10</td>
<td>( Q_1 )</td>
<td>64.37</td>
<td>( A_1 )</td>
<td>24.78</td>
</tr>
<tr>
<td>( \alpha_2 )</td>
<td>0.03896</td>
<td>( n_2 )</td>
<td>45.98</td>
<td>( Q_2 )</td>
<td>-6593.2</td>
<td>( A_2 )</td>
<td>-159.7</td>
</tr>
<tr>
<td>( \alpha_3 )</td>
<td>-0.07531</td>
<td>( n_3 )</td>
<td>-101.84</td>
<td>( Q_3 )</td>
<td>2836.90</td>
<td>( A_3 )</td>
<td>478.36</td>
</tr>
<tr>
<td>( \alpha_4 )</td>
<td>0.06256</td>
<td>( n_4 )</td>
<td>-3.37</td>
<td>( Q_4 )</td>
<td>-2650.3</td>
<td>( A_4 )</td>
<td>-373.5</td>
</tr>
<tr>
<td>( \alpha_5 )</td>
<td>-0.02090</td>
<td>( n_5 )</td>
<td>2.36</td>
<td>( Q_5 )</td>
<td>1253.63</td>
<td>( A_5 )</td>
<td>243.26</td>
</tr>
</tbody>
</table>

Fig. 6 - The variation of material constants as a function of strain (a) \( \alpha \) vs true strain, (b) \( n \) vs true strain, (c) \( \ln (A) \) vs true strain, and (d) \( Q \) vs true strain.
Fig. 7 — Comparison between predicted and measured flow stress curves of aluminum alloy at strain rates of (a) 0.001 s$^{-1}$, (b) 0.01 s$^{-1}$ and (c) 0.1 s$^{-1}$.

Fig. 8 — Correlation between the experimental and predicted flow stress data from the developed constitutive equation.

4. Conclusion

In this paper, a set of predetermined hot compression tests were conducted and reported for an extruded aluminum alloy. The constitutive equation to predict flow stress for a given strain was derived using the experimental data. Further, the validity of the developed equation considering strain compensation had been verified by comparing the predicted and experimental results. The following are the conclusions arrived.

- The true stress–true strain curves had shown significant dependence of the flow stress on deformation strain rate and temperature. The curve exhibits a slight dip in flow stress after the peak value at 673 and 773 K. Whereas at 373, 473 and 573 K, after the initial rise, the flow stress gently approaches a supposition value.
- Microstructural investigation revealed relatively equiaxed and defect-free grains with high angle grain boundaries in the case of extruded aluminum alloy compared to as-cast alloy.
- The material constants are found to be varying as functions of strain and the dependency is found to be a polynomial relation of fifth order.
- The developed deformation constitutive equation incorporating the compensation of strain is found to provide an accurate estimate of the flow stress for the considered zinc-based aluminum alloy with a correlation coefficient of 0.9965 and average absolute relative error of 4.62%.

Conflicts of interest

The authors declare no conflicts of interest.

REFERENCES


